PRACTICAL SANITARY SCIENCE

PART II.

CALCULATION OF AREAS, CUBIC SPACE, &C.

W. H. MAXWELL

Formerly

Presented by

The author

ber 1897

T… SOCIETY
…OMOTION
O… …TH

…rrowed by
personally or
erson to whom
…pt for the…

…e n…

…a
…e …
…il

NOTES

ON

PRACTICAL SANITARY SCIENCE

BY

WILLIAM H. MAXWELL,

ASSISTANT ENGINEER AND SURVEYOR, LEYTON URBAN DISTRICT COUNCIL,
AUTHOR OF "DRAINAGE WORK AND SANITARY FITTINGS,"
AND "VENTILATION, HEATING, AND LIGHTING,"
CERTIFICATES OF THE ASSOCIATION OF MUNICIPAL AND COUNTY
ENGINEERS AND SANITARY INSTITUTE.

2/2

PART II.

CALCULATION OF AREAS, CUBIC SPACE, &c.

Being a Summary of the Subject of Mensuration for the Use of Engineers, Surveyors, &c.

WITH NUMEROUS ILLUSTRATIONS.

(*Reprinted from* The Sanitary Record.)

LONDON:
THE SANITARY PUBLISHING COMPANY, LTD.,
5, Fetter Lane, Fleet Street, E.C.
1897.

CONTENTS.

Part II.

CALCULATION OF AREAS, &c.

PREFACE.

THE object of these "Notes" is to briefly consider, mainly from a Sanitary Engineer's point of view, a few of the more important matters connected with the practical side of a somewhat comprehensive subject, viz., Sanitary Science.

The Notes are proposed to be given under the heads of Ventilation, Heating, and Lighting; the Calculation of Areas and Cubic Space; the Removal and Disposal of the Refuse of Towns; Water and Water Supply; and Plumber's Work; the present book forming the second part of the Series.

It is, of course, not intended to deal in anything like an exhaustive manner with the subjects mentioned (any *one* of which may form the subject of a large volume, or be the individual work of a specialist), but principally to give such information in connection therewith as will be of practical interest and use to those whose duties or interests lie within this sphere.

A large part of the "Notes" were originally prepared by the Author, from time to time, during the course of his own experience and studies, and, at the request of the publishers, have since been somewhat amplified and illustrated throughout, and are here given with the special hope that they may be of service to candidates preparing for professional certificates (now almost a *sine quâ non*), such, for example, as those granted by the Association of Municipal and County Engineers and the Sanitary Institute.

The present Part, consisting as it does of a large number of

Rules, Figures, Formulæ, &c., which, as a whole, are seldom retained *with certainty* for very long in the memory, will be found, it is believed, a handy summary of the subject of *Mensuration* for the reference and use of Engineers and Surveyors generally.

William H. Maxwell.

Town Hall, Leyton, E.,
1897.

ON THE

CALCULATION OF AREAS, CUBIC SPACE, &c.

ON THE

CALCULATION OF AREAS, CUBIC SPACE, &c.

CHAPTER I.

THE calculation of *lengths*, *areas*, and *volumes* from a few given data, or that branch of Applied Geometry known as *Mensuration*, is one of such numerous and important applications to all sorts of practical questions arising in the every-day life of the Surveyor, that a brief outline of the subject for the interest of those requiring it, presenting a few of the more important geometrical facts and rules, will now be given as a part of these "Notes."

Mensuration may be looked upon as a combination of Arithmetic and Geometry, and in it many of the principal facts of Geometry are introduced and usefully *applied* to practical calculations; such, for example, as the calculation of the cubic capacity and floor area of variously-shaped rooms, the contents of vessels, the heights of spires or shafts, the areas of irregularly-shaped fields, &c. &c.; in fact, the whole operation of "Land Surveying" is *based* entirely upon a knowledge of this subject.

It need scarcely be mentioned that it is, of course, assumed that the reader is already quite familiar with the usual signs and abbreviations used in mathematics, with the elements of Arithmetic, including the extraction of the square ($\sqrt{}$) and cube ($\sqrt[3]{}$) roots, and also with the usual *definitions* in Euclid's Elements of a "point," a "line," the various kinds of "angles," of "triangles," and of four-sided figures, &c. &c. Before proceeding further, however, it will be advantageous to refresh the mind with a few of the most important *theorems* as *demonstrated* in Euclid's Elements, a knowledge of which is essential, even for the most superficial study of this subject. In fact, if the *geometrical principles involved* are first thoroughly understood, the *application* of the various "Rules" which will be given to any practical cases which may arise, becomes a very simple matter, and the calculation is resolved into a mere arithmetical operation.

GEOMETRICAL THEOREMS.

I.—If the straight line A B makes with the straight line C D, on one side of it, the angles C B A and A B D; then these angles will

be together equal to two right angles. (For demonstration see Euclid I. 13).

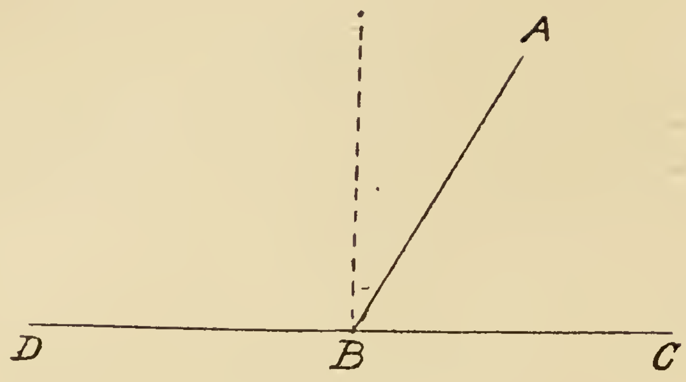


Fig. 1.

II.—If two straight lines A B and C D cut one another at E, then the angle A E C will be equal to the angle B E D, and the angle A E D to angle B E C. (Euclid I. 15.)

The angles A E C and B E D are called "*vertically opposite*" angles; and from this proposition it is clear that all the angles at

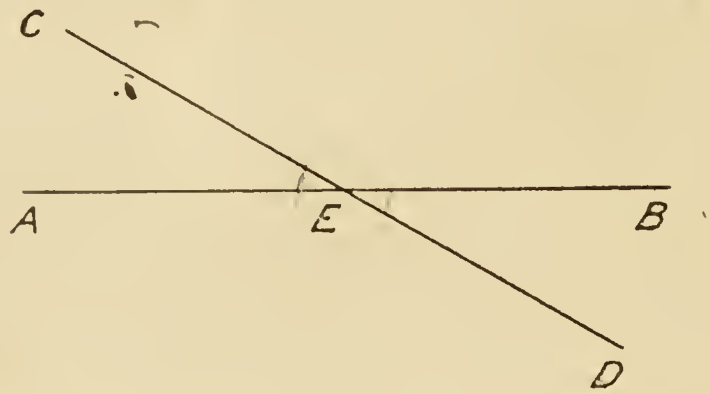


Fig. 2.

point E are together equal to four right angles, and that all the angles made by any number of straight lines meeting at one point, as E, are together equal to four right angles.

III.—If a straight line E F falls on the parallel straight lines A B,

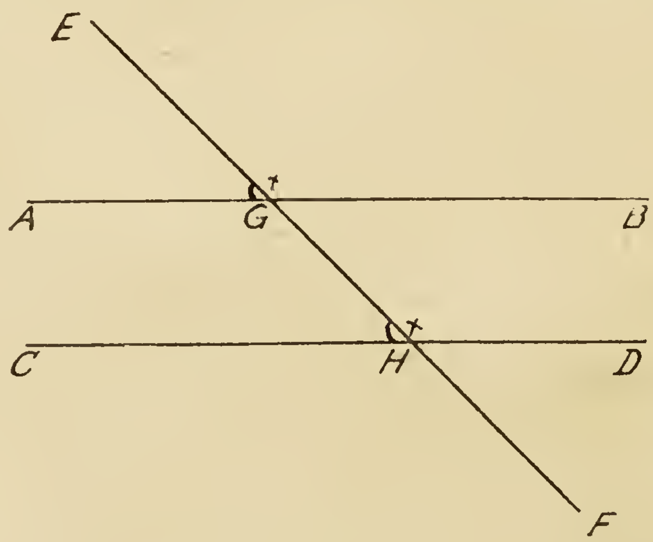


Fig. 3.

C D; then the angle E G B will be equal to the angle G H D, and

the angles B G H and G H D will be together equal to two right angles. (Euclid I. 29.)

The angles A G H and G H D are known as "*alternate angles,*" and are equal to each other, A B being parallel to C D.

IV.—Let the side B C of the triangle A B C be produced to D; then the exterior angle A C D will be equal to the two interior and opposite angles A B C and B A C. (Euclid I. 32.)

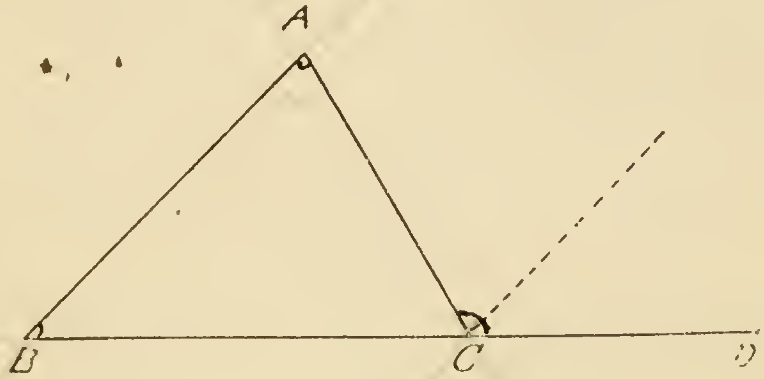


Fig. 4.

Also, the three interior angles of every triangle are together equal to two right angles.

And, all the interior angles of any rectilineal figure, together with four right angles, are equal to twice as many right angles as the figure has sides.

And, all the exterior angles of any rectilineal figure are together equal to four right angles.

V.—If two sides A B, A C of a triangle A B C are equal, the angles A B C and A C B opposite to them will also be equal. (Euclid I. 5.)

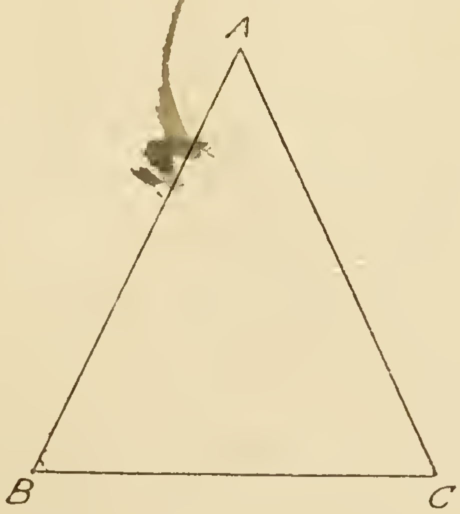


Fig. 5.

VI.—If the two angles A B C, A C B (Fig. 5) are equal, then the sides A B, A C which are opposite to them will also be equal. (Euclid I. 6.)

VII.—Any two sides of a triangle are together greater than the third side. (Euclid I. 20.)

VIII.—If the two sides A B, A C of one triangle, are equal to two sides D E, D F of another triangle, each to each, and the angle B A C contained by the two sides of the one equal to the angle

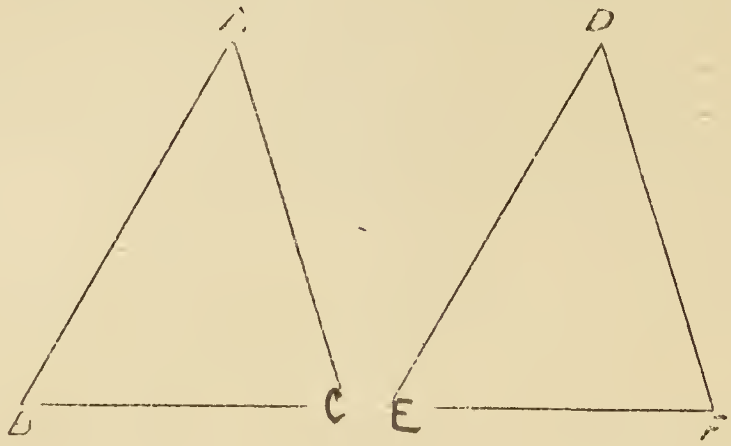


FIG. 6.

E D F contained by the two sides of the other, the triangles will be equal in all respects. (Euclid I. 4.)

IX.—If two angles A B C, A C B of one triangle are equal to two angles D E F, D F E of another triangle, each to each, and the side

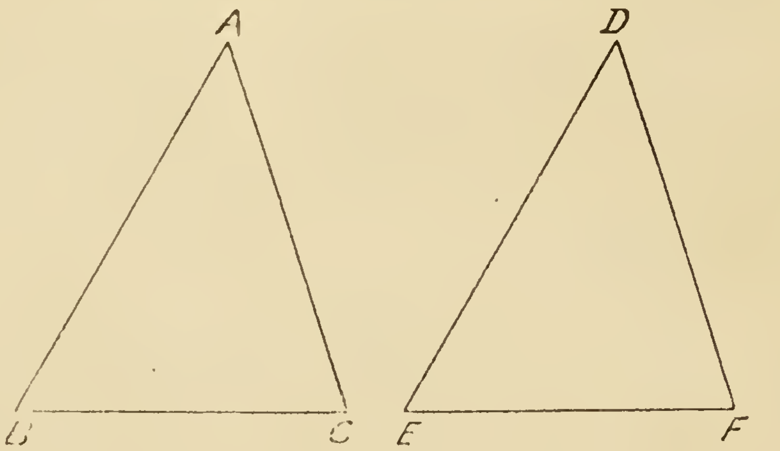


FIG. 7.

B C adjacent to the two angles of the one equal to the side E F adjacent to the two angles of the other, the triangles will be equal in all respects. (Euclid I. 26.)

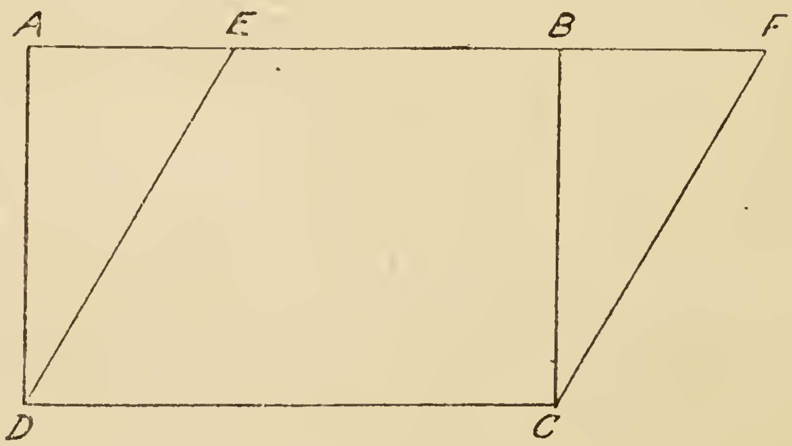


FIG. 8.

X.—A parallelogram C D E F is equivalent to a rectangle A B C D

on the same base and between the same parallels (*i.e.*, of the same height). (Euclid I. 36.)

XI.—If a rectangle A E F C and a triangle A B C are on the

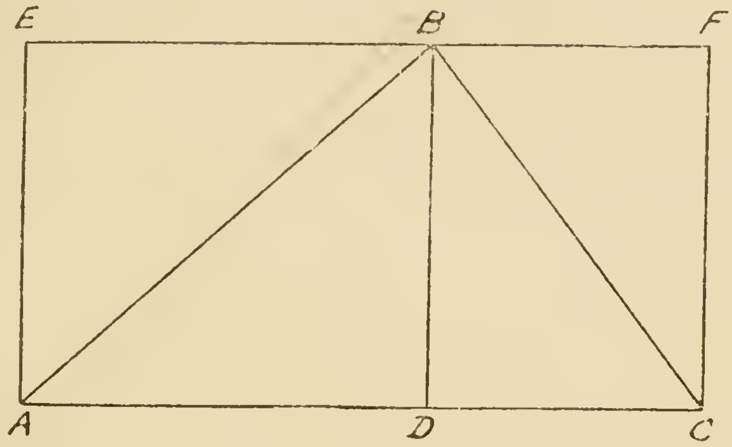


Fig. 9.

same base and of the same height, the rectangle will be double of the triangle. (Euclid I. 41.)

XII.—In any right-angled triangle A B C the square described on the hypotenuse A C is equal to the sum of the squares described on the sides A B and B C.

This statement is one of the most important and useful in geometry; it is said to have been discovered by Pythagoras, a celebrated

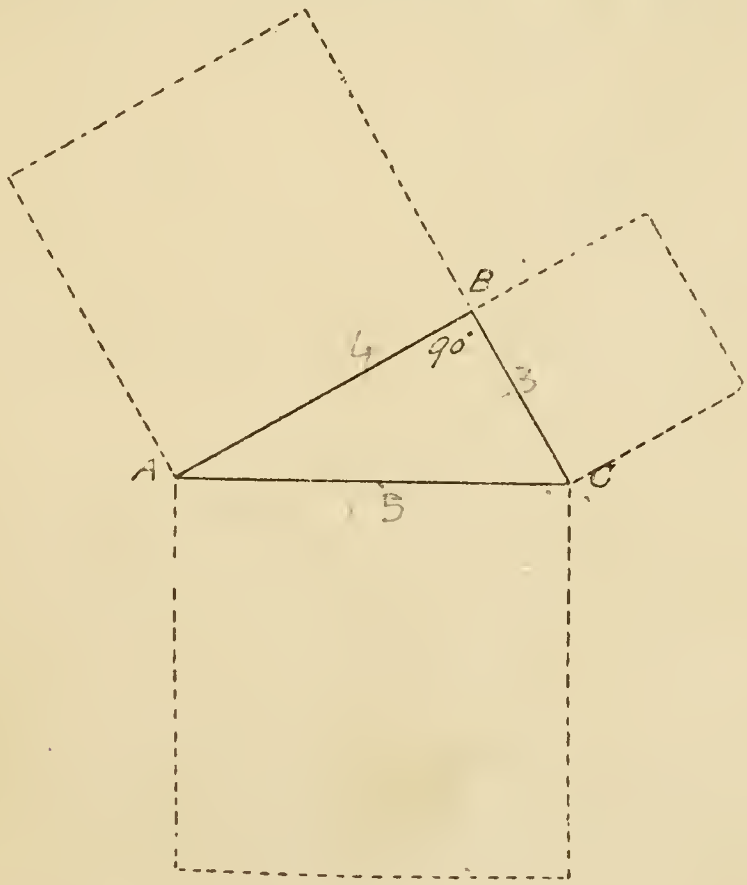


Fig. 10.

Greek philosopher, who was born about 2400 years ago. Where any two sides of a right-angled triangle are given, the third can be easily calculated, as—

$$A\,C = \sqrt{A\,B^2 + B\,C^2}.$$

XIII.—If a straight line as A B touch a circle, the radius C D drawn to the point of contact D will be perpendicular to the straight line. (Euclid III. 18.)

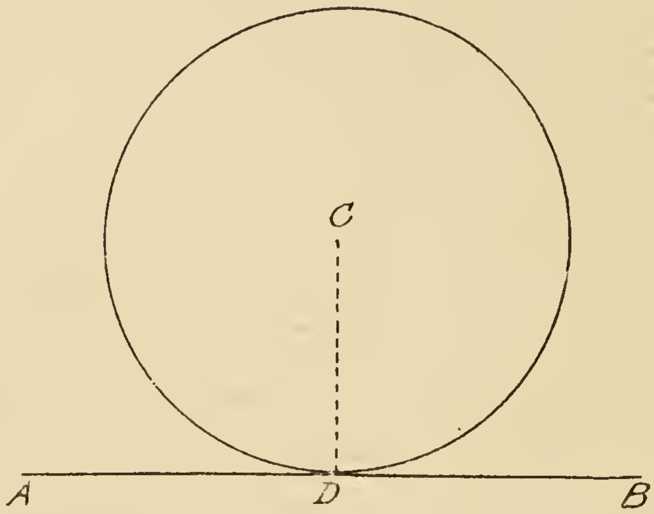


Fig. 11.

Note.—A straight line is said to *touch* a circle, when it meets the circle, and being produced, does not cut it.

XIV.—Angles, as A B C and A D C, in the same segment, as A B D C, of a circle, are equal to one another. (Euclid III. 21.)

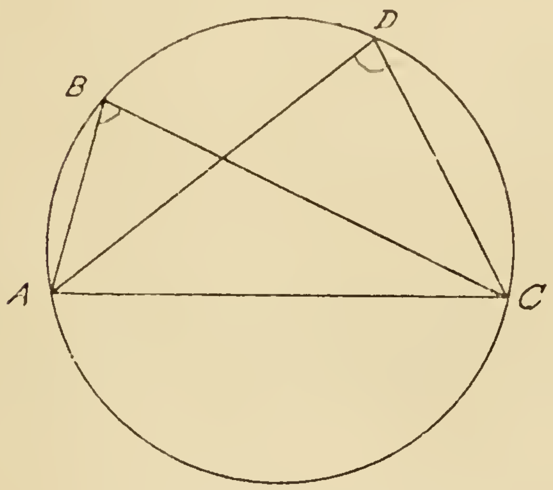


Fig. 12.

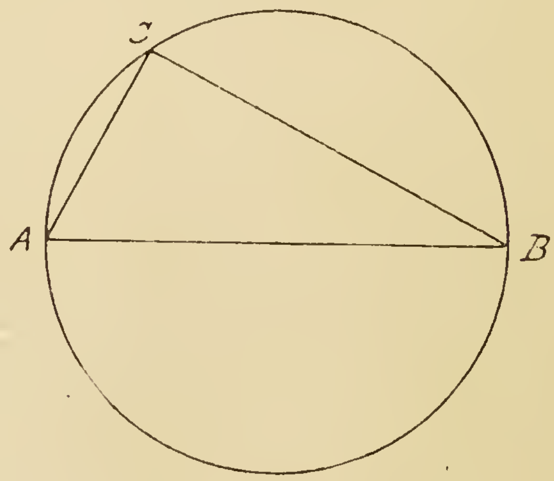


Fig. 13.

XV.—A B being the diameter of a circle, and C any point in the circumference, the angle formed by lines A C, C B is a right angle; but the angle in a segment greater than a semicircle is less than a right angle; and the angle in a segment less than a semicircle is greater than a right angle. (Euclid III. 31.)

XVI.—If the triangles A B C and A C D, and the parallelograms E C and C F have the same altitude, then, as the base B C is to

the base C D, so is the triangle A B C to the triangle A C D, and the parallelogram E C to the parallelogram C F. (Euclid VI. I.)

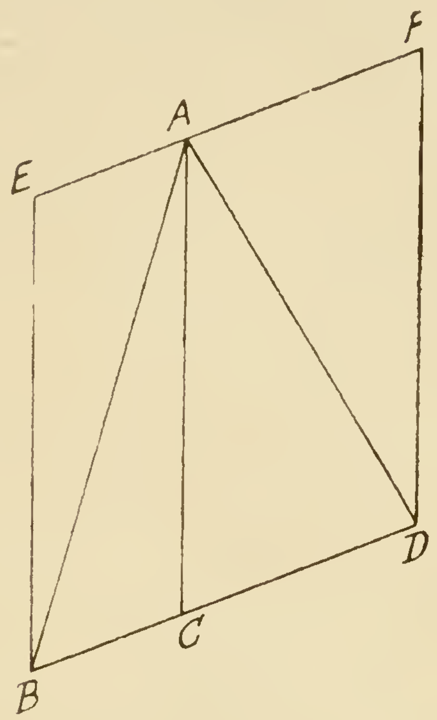


Fig. 14.

XVII.—If in any triangle A B C, D E be drawn parallel to one of its sides C B; then A B : A E :: B C : D E (Euclid VI. 2), and the two triangles A C B and A D E are said to be "*similar*."

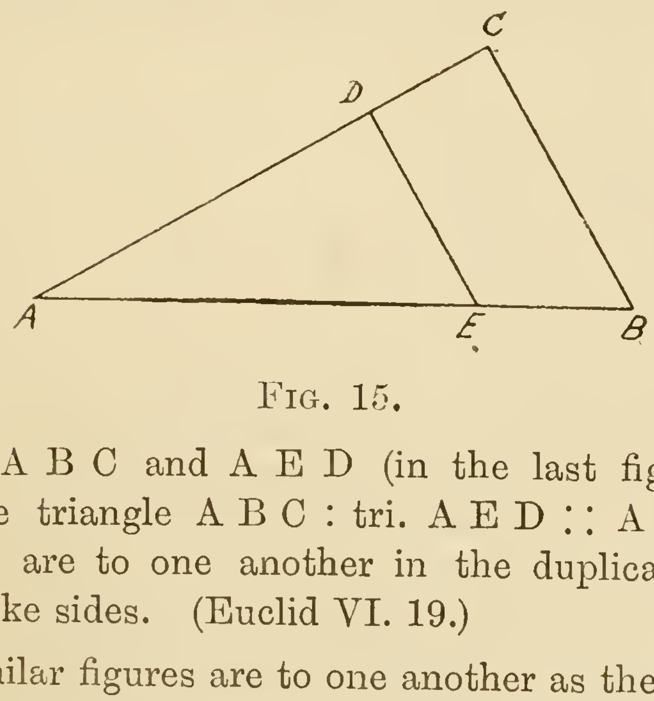


Fig. 15.

XVIII.—Let A B C and A E D (in the last figure) be *similar* figures then the triangle A B C : tri. A E D :: A B^2 : A E^2 *i.e.*, similar triangles are to one another in the duplicate ratio of their homologous or like sides. (Euclid VI. 19.)

XIX.—All similar figures are to one another as the squares of their homologous or like sides. (Euclid VI. 20.)

Note.—If in two similar triangles one side is three times the length of the corresponding side of the other, the area of the larger triangle is nine times the area of the smaller.

XX.—The areas of circles are to one another as the squares of their diameters. (Euclid XII. 2.)

Note.—All circles are similar figures.

XXI.—All similar solids are to one another as the cubes of their like linear dimensions.

XXII.—A B and C D are two chords of a circle meeting at E join B C and A D; then the triangles A E D and B E C will be

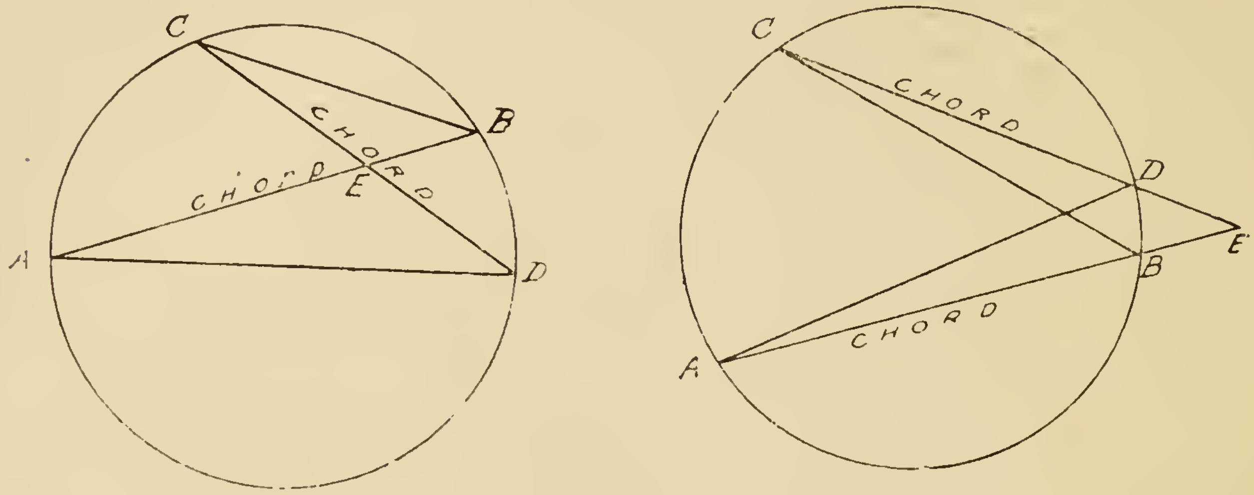


Fig. 16.

similar—the angles E A D and E C B being equal, and the angles E D A and E B C being equal. (See theorem XIV.)

ON THE CALCULATION OF LENGTHS.

Although the following table of measures of length will be familiar to all, it will be convenient to give it here in the following form:—

Inches.		Feet.		Yards.		Poles.		Furlongs.		Mile.
12	=	1		—		—		—		—
36	=	3	=	1		—		—		—
198	=	$16\frac{1}{2}$	=	$5\frac{1}{2}$	=	1		—		—
7,920	=	660	=	220	=	40	=	1		—
63,360	=	5280	=	1760	=	320	=	8	=	1

In "Land Surveying" the standard of measurement is the "*Chain.*" That in general use is what is known as *Gunter's Chain*, which is 66ft. long, and is divided into 100 links of $\left(\frac{66 \times 12}{100}\right)$ 7·92in. each. A strip of land 10 chains long and 1 chain wide is one acre; also 10 chains = 1 furlong, as is shown in the above table.

In practice, it may be taken as a general rule that in all questions in Mensuration, if the figure under solution can be accurately plotted to a fairly large scale of equal parts it will greatly assist in *speedily* arriving at the required lengths or areas; but, at the same time it is equally essential that the proper rules, including the necessary Geometry, should be fully understood, so as to enable the entire calculation being made mathematically without the aid of "plotting" and measuring by the use of the scale.

RIGHT-ANGLED TRIANGLES.

When two sides of a right-angled triangle are known, the third can easily be calculated, the rule for which depends on theorem No. XII. (ante).

Given the sides of a right-angled triangle, find the hypotenuse :—

(1) *Add the squares of the sides and extract the square root of the sum.*

Given the hypotenuse and one side of a right-angled triangle, find the other side :—

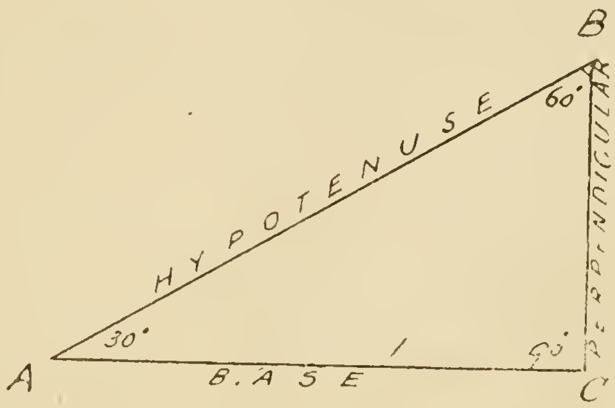


FIG. 17.

(2) *From the square of the hypotenuse subtract the square of the given side, and extract the square root of the remainder.*

(3) Or another rule is—*Multiply the sum of the hypotenuse and the side by their difference, and extract the square root of the product.*

The rules (Nos. 1, 2, 3) given relating to the right-angled triangle (see Fig. 17) may be briefly stated in the following form :—

$$A\,B = \sqrt{A\,C^2 + C\,B^2}$$

$$A\,C = \sqrt{A\,B^2 - B\,C^2}$$

$$C\,B = \sqrt{A\,B^2 - A\,C^2}$$

[1] "When the area of a right-angled triangle and the hypotenuse are given, the legs may be found by the following general rule :—To the square of the hypotenuse add four times the area of the triangle, and the square root of this number will be the sum of the legs. From the square of the hypotenuse take four times the area of the triangle, and the square root of the remainder will be the difference of the legs. Add half the difference of the legs to half their sum, and you will obtain the *greater* leg; but if half the difference of the legs be taken from half their sum, the remainder will be the less leg."

The Use of Trigonometry and Logarithms.

Every plane triangle consists of six parts, viz., three angles and three sides, and whenever three of these parts are known, one of

1 "Nesbit's Practical Mensuration." (Longmans, Green, and Co.)

which must be a side, the remaining parts can be calculated. This process is known as the "*solution of triangles*," and comes under the head of TRIGONOMETRY, in the operations of which the numerical calculations which occur are greatly abbreviated by the aid of "LOGARITHMS." These subjects are of great *practical* utility, and are necessary for the complete solution of the triangle; they afford special service in calculations in "Land Surveying," and require to be studied in works specially devoted to them.

By means of a table of Logarithms the *multiplication* of numbers is accomplished by the addition of their logarithms; *division* by their subtraction; *involution* or raising of powers by their multiplication by the index; and *evolution* or extraction of roots by their division by the radix. The method of working by the aid of logarithms is comparatively simple, and may be applied to almost all kinds of calculations. As an illustration of the application of logarithms, some of the "practical examples," to be given later, have been worked out by their aid. Should the reader, however, not be already familiar with their use, he is recommended to study the few introductory pages of a capital little table of "Five-figure Logarithms," compiled by Mr. C. J. Woodward, B.Sc.[1]

An important trigonometrical function in connection with the solution of triangles has also been introduced into the working of some of

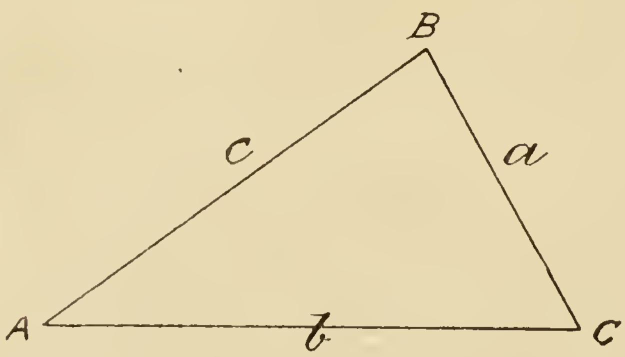


FIG. 18.

the "examples," viz., that *the sides of a triangle are proportional to the sines of the opposite angles;* for example, in a triangle A B C, whose sides are represented by a, b, and c respectively, the sine of the angle at A is to the side a as the sine of the angle at B is to the side b, that is sin A : a :: sin B : b.

The term "*sine*" means the length of a perpendicular B C (Fig. 19) drawn from one extremity B of an arc B D of a circle to the radius A D drawn through the other extremity D. Thus, B C is the sine of the arc B D and of the angle B A C, the radius A D of the circle

[1] Published by Simpkin, Marshall, and Co., London.

being *unity*. In a right-angled triangle the *sine* is the side opposite the given angle divided by the hypotenuse, *i.e.*,

$$\frac{\text{Perpendicular}}{\text{Hypotenuse}} = \text{sine}.$$

The above-mentioned property is a very useful one, and can be

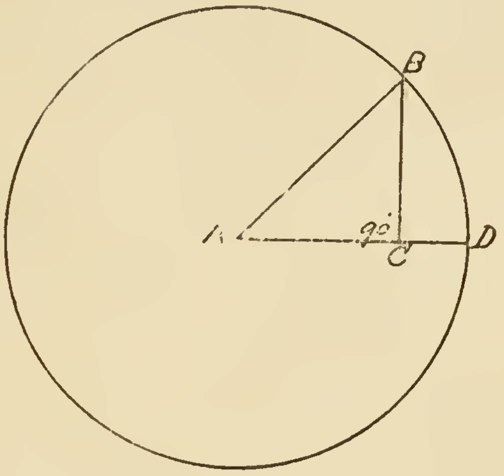


Fig. 19.

extensively applied to the solution of many problems arising in connection with the subject of *Mensuration*.

Similar Figures.

Questions arising in connection with similar triangles as A B C and D E F often occur in practice.

If two sides as C B and B A of a triangle are known, and one of the like sides of a similar triangle, the other corresponding side of

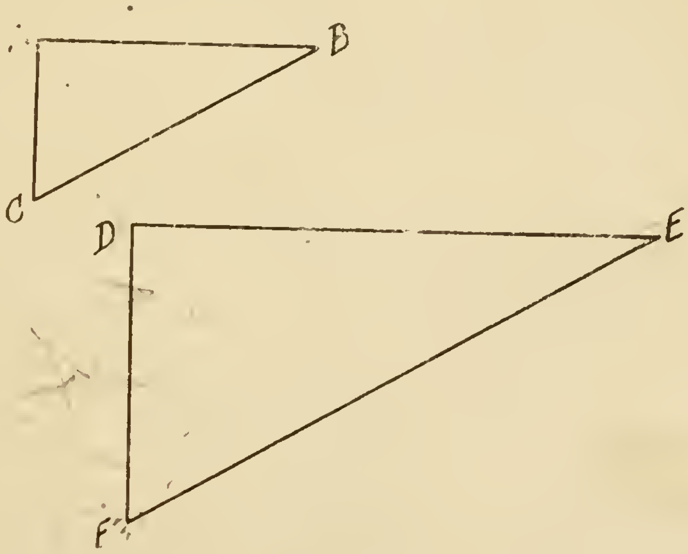


Fig. 20.

the second triangle can be calculated by the "Rule of Three," for

C B : B A :: F E : E D.

All equilateral triangles are similar figures; and the base of an equilateral triangle is to its height as 1 is to ·866..., so that the

height of these figures can always be calculated as follows :—Base $\times$ ·866 = height, and, where x = base, the Area = $x \times \frac{1}{2}$ (·866 x).

In the figures accompanying theorem XXII., the triangles A E D and B E C are *similar*, therefore—

$$\text{E A} : \text{E D} :: \text{E C} : \text{E B} \therefore \text{E A} \times \text{E B} = \text{E D} \times \text{E C}$$

which is a very important property of the circle.

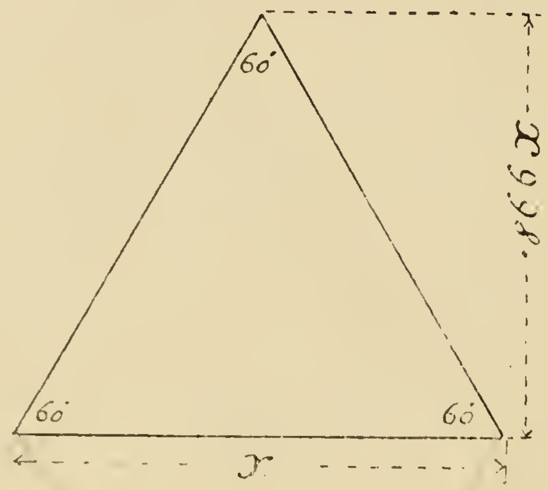


Fig. 21.

By the aid of similar triangles and the rule of proportion, the height of an object can be calculated from the length of its shadow as compared with that of a smaller object, of which the shadow and height can both be measured.

Chords of a Circle.—A B is the chord of a circle, whose centre is D; F C is perpendicular to A B, and bisects it at E. A B is the "chord

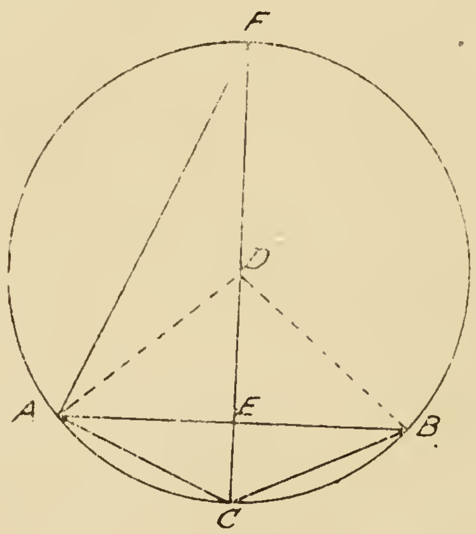


Fig. 22.

of the arc" A C B, whilst A C or C B is the "chord of half the arc," and C E is the "height of the arc."

The triangles C A F and C E A are similar, the angle C A F being a right angle (theorem XV.), and C E : C A :: C A : C F.

$$(a) \therefore \text{C E} \times \text{C F} = \text{C A} \times \text{C A} = \text{C A}^2.$$

And, from what has just been said above in connection with theorem XXII.—

$$(b)\ C E \times E F = A E \times E B = A E^2.$$

On these two statements depend the following rules as to chords of a circle.

To find the diameter of the circle, the height of an arc and the chord of half the arc being given—

(4) "*Divide the square of the chord of half the arc by the height of the arc, and the quotient will be the diameter of the circle.*"

For from statement (*a*) above—

$$C E \times C F = C A^2.$$

$$\therefore \quad C F = \frac{C A^2}{C E}.$$

To find the height of the arc, the chord of half an arc and the diameter of circle being given—

(5) "*Divide the square of the chord of half the arc by the diameter of the circle, and the quotient will be the height of the arc.*"

$$\because C E \times C F = C A^2.$$

$$\therefore \quad C E = \frac{C A^2}{C F}.$$

To find the chord of half the arc, the height of an arc and the diameter being given—

(6) "*Multiply the diameter of the circle by the height of the arc; the square root of the product will be the chord of half the arc.*"

$$\because C E \times C F = C A^2.$$

$$\therefore \quad C A = \sqrt{C E \times C F.}$$

To find the diameter of the circle, the chord of an arc and the height of the arc being given—

(7) "*Divide the square of half the chord by the height, and the quotient will be the remaining part of the diameter; so that the sum of the quotient and the given height will be the diameter.*"

For from statement (*b*) above—

$$C E \times E F = A E^2.$$

$$\therefore \quad E F = \frac{A E^2}{C E}.$$

$$\therefore E F + C E \text{ (the given height)} = \text{Diameter.}$$

The Circumference of a Circle.—To find the circumference of a circle, the diameter being given—

(8) *Multiply the diameter by* 3·1416.

The proportion between the diameter and circumference of a circle is approximately as 1 is to 3·1416, but the *exact* relation cannot be

given; the calculation has been carried to more than 600 places of decimals, but the error in the above rule is at the rate of less than one foot per 75 miles.

The Greek letter π (pi) is generally used to denote the number $3 \cdot 1416$.

The difference of the diameters of any two circles, multiplied by $3 \cdot 1416$, equals the difference of their circumference.

Chords of a circle equally distant from its centre are equal to one another.

A *tangent* to a circle is a line perpendicular to the extremity of its radius.

To find the diameter of a circle, the circumference being given—

(9) *Divide the circumference by* $3 \cdot 1416$.

The Arc of a Circle.—B C is an arc of a circle, of which A is the centre, and the angle B A D a right angle.

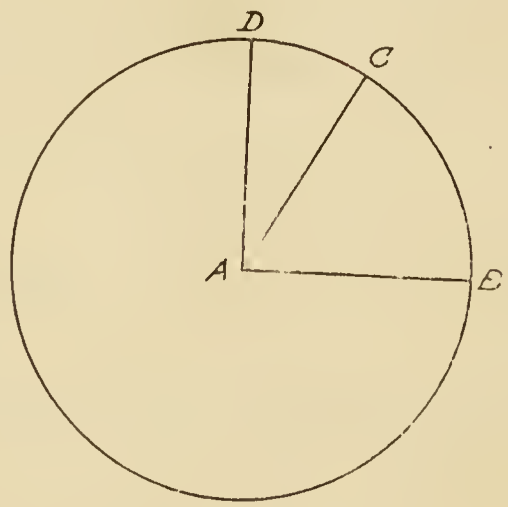


Fig. 23.

B C : B D :: angle B A C : angle B A D ∴ B C : circumference of circle :: angle B A C : four right angles, or 360 degrees.

To find the length of an arc, the number of degrees in the angle subtended by the arc being given—

(10) 360 : *number of degrees in angle* :: *circumference* : *length of arc.*

To find number of degrees in the angle subtended by an arc, the length of arc being given—

(11) *Circumference of circle* : *length of arc* :: 360 : *number of degrees in the angle.*

To find the length of an arc, the chord of the arc, and the chord of half the arc being given—

(12) *From* 8 *times the chord of half the arc subtract the chord of the whole arc, and divide the remainder by* 3.

SUMMARY OF PROPERTIES OF THE CIRCLE.

At this stage it may perhaps be convenient to give a summary of the *properties of the circle*, including, for the sake of completeness, references to its *area*, as well as circumference and diameter:—

Circumference = diameter × 3·1416
,, = radius × 6·28318
,, = $\sqrt{\text{area}}$ × 3·5449
Diameter = circumference × ·31831
,, = $\sqrt{\text{area}}$ × 1·1283
Radius = circumference × ·1591
,, = $\sqrt{\text{area}}$ × ·564
Area = diameter^2 × ·7854
,, = radius^2 × 3·1416
,, = circumference × diameter × ·25
Length of arc = number of degrees × ·017453[1] radius.
Side of inscribed square = diameter × ·7071.
Side of equal square = diameter × ·886226.
Diameter of circle of equal area to square = side of square × 1·1283.
Side of a square × 1·414214 = diameter of its circumscribing circle.
Side of a square × 4·442883 = circumference of its circumscribing circle.
Side of a square × 3·5449 = circumference of circle equal in area.
The area of a circle = the rectangle of its radius and semi-circumference.
The area of a circle = area of a triangle whose base equals the circumference and perpendicular the radius.
A square circumscribed about a circle = four times the square of the radius; also, a square inscribed in a circle = half the circumscribed square.
Similar figures inscribed in circles are to each other as the squares of the diameters of the circles.
The circle is the figure enclosing the largest area for a given perimeter.

ON THE CALCULATION OF AREAS.

For convenience, the table of square measure is given here in the following form:—

Inches.		Feet.		Yards.		Perches.		Roods.		Acre.
144	=	1		—		—		—		—
1,296	=	9	=	1		—		—		—
39,204	=	272¼	=	30¼	=	1		—		—
1,568,160	=	10,890	=	1,210	=	40	=	1		—
6,272,640	=	43,560	=	4,840	=	160	=	4	=	1

10 square chains = 1 acre.

---

[1] Arc of 1 deg. to radius 1 = ·017453. If radius is 1, diameter = 2, and circumference = 2 × 3·1416 = 6·2832, and $\frac{6\cdot2832}{360 \text{ degs.}}$ = ·017453.

To find the area of a rectangle—

(13) *Length* × *breadth* = *area.*

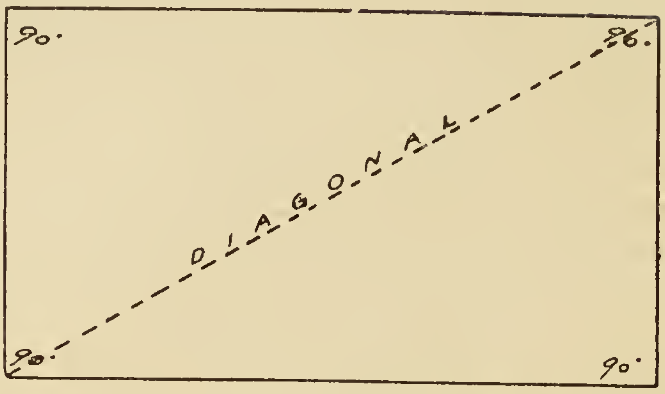


Fig. 24.

Note :—The *diagonal* of a rectangle is equal to the square root of the sum of the squares of any two adjacent sides.

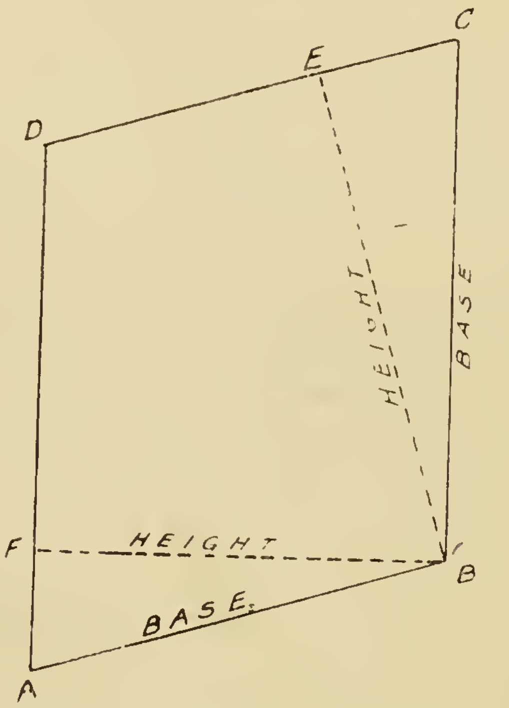
To find the length of a side of a square whose area is given,—

(14) $\sqrt{area}$ = *length of a side.*

Note :—The *diagonal* of a square is, of course, equal to the square root of twice the square of a side. To find the area of a parallelogram—

(15) *Base* × *height* = *area.*

As, A B × B E = area.
or B C × B F = area.



Opposite Sides Parallel and Equal.

Fig. 25.

Note :—The area of a parallelogram may also be found by multiplying the product of any two of its sides by the natural sine of the included angle.

To find the area of a triangle—

(16) *Multiply the base by the perpendicular height and divide by 2.*

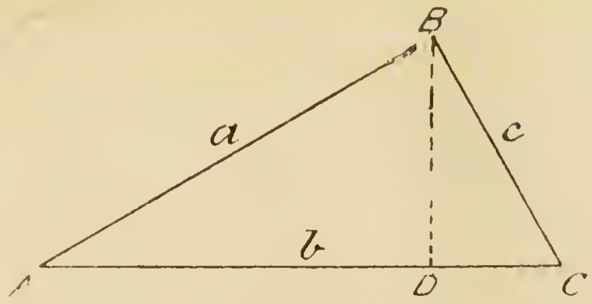


Fig. 26.

To find area of triangle when the three sides only are known—

(17) *From half the sum of the three sides subtract each side separately; multiply the half sum and the three remainders together; the square root of the product is the area.*

Let s = the $\frac{1}{2}$ sum of the three sides, and a b c = the three sides respectively, then the above rule may be expressed as follows:—

$$\sqrt{s\ (s-a)\ (s-b)\ (s-c)} = \text{area}.$$

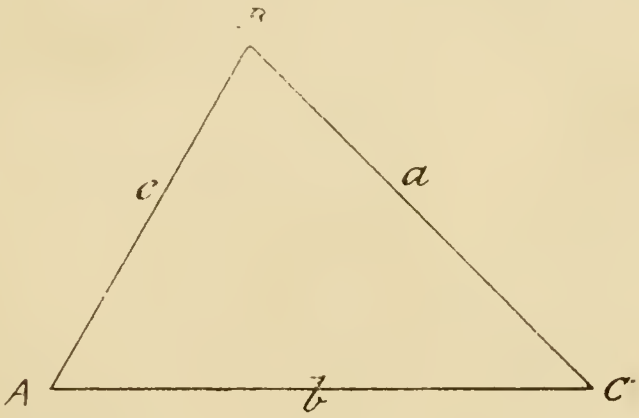


Fig. 27.

Note:—Divide the area of a triangle by half the sum of the sides, and the quotient will be the radius of the inscribed circle.

Also, the *area of any plane triangle* A B C (Fig. 27)—

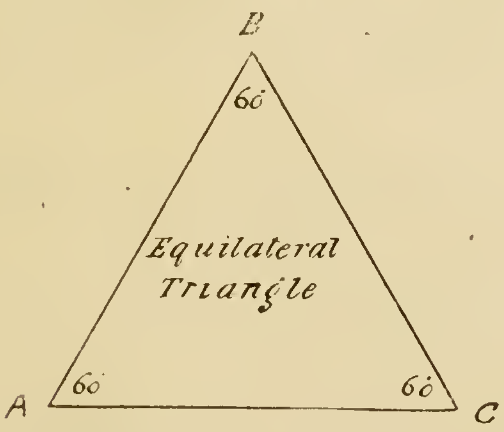


Fig. 28.

$$= \frac{b\ c\ \sin A}{2}$$

$$= \frac{a\ b\ \sin C}{2}$$

$$= \frac{a\ c\ \sin B}{2}$$

and in the case of an *equilateral triangle* (Fig 28), the area = $\frac{1}{2}$ A C^2 sin B A C.

The *sines* of some important angles may here be conveniently stated in the following form—

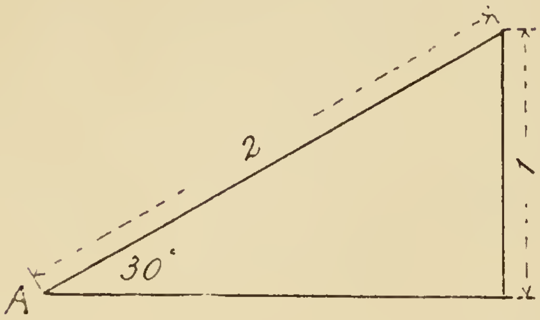


Fig. 29.

Sin A $= \frac{1}{2} =$ ·5.

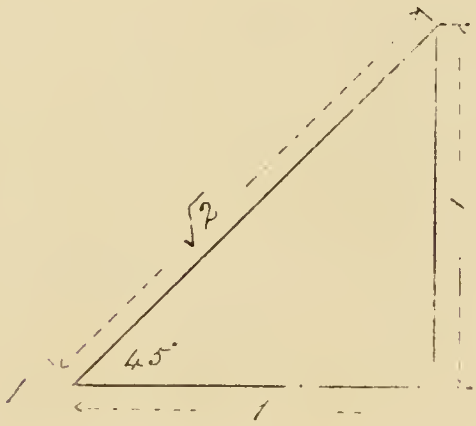


Fig. 30.

Sin A $= \frac{1}{\sqrt{2}} =$ ·7071068.

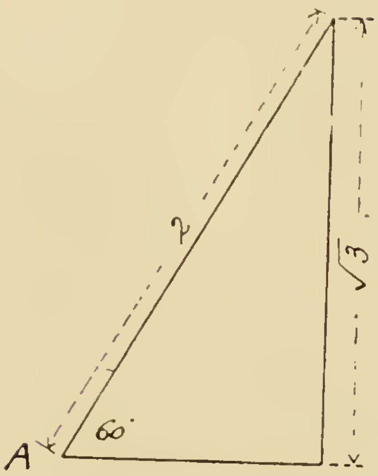


Fig. 31.

Sin A $= \frac{\sqrt{3}}{2} =$ ·8660254.

The sine of an angle of 90 deg. = 1, and since with angles between 90 deg. and 180 deg., sin A = sin (180 deg. − A), therefore the

Sine of an angle of 120 deg. = sin (180 − 120) = sin < 60 deg. as above.

Sine of an angle of 135 deg. = sin (180 − 135) = sin < 45 deg. as above.

Sine of an angle of 150 deg. = sin (180 − 150) = sin < 30 deg. as above.

To find the diameter of the circle described round a triangle whose sides are given—

(18) *The diameter is equal to the product of the sides of the triangle divided by twice the area of the triangle.*

To find the area of a trapezoid—

(19) *Multiply the sum of the two parallel sides by the perpendicular distance between them, and half the product is the area.*

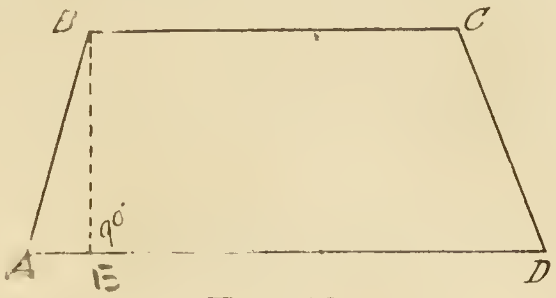


FIG. 32.

That is, $\frac{1}{2}$ (A D + B C) × E B = area.

To find the area of any rectilineal figure—

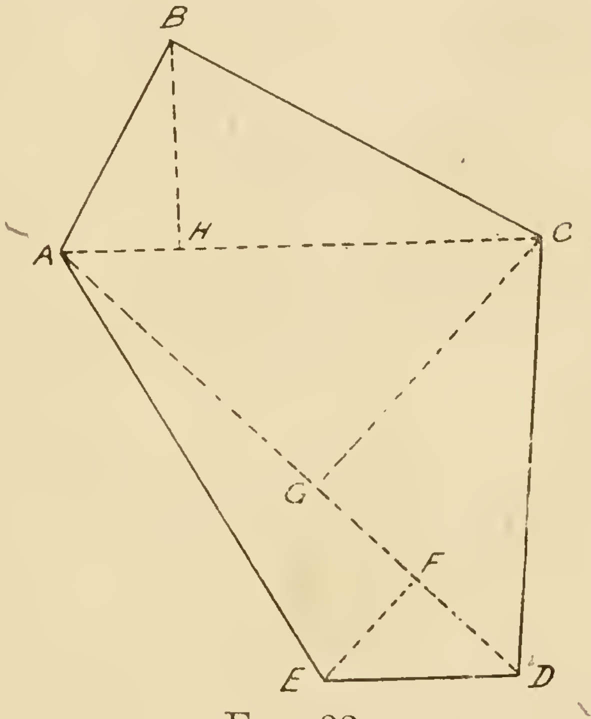


FIG. 33.

(20) *Divide the figure into triangles by lines from its angles; calculate the area of each triangle and add the results.*

$$\text{Area} = \tfrac{1}{2}\,(\text{A C} \times \text{H B}) + \tfrac{1}{2}\,(\text{A D} \times \text{F E}) + \tfrac{1}{2}\,(\text{A D} \times \text{G}$$

$$= \frac{(\text{A C} \times \text{H B}) + \text{A D}\,(\text{F E} + \text{G C})}{2}$$

To find the area of a regular polygon—

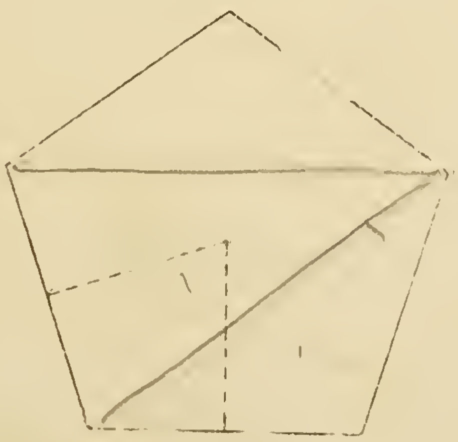


FIG. 34.

(21) *Multiply the length of a side by the perpendicular height to the centre, and by the number of sides; half the product will be the area.*

Note:—The side of a *hexagon* is equal to the radius of the circumscribed circle.

To calculate the length of a side of an octagon, the diameter D being given—

(22) *Multiply the diameter by* ·4142136.

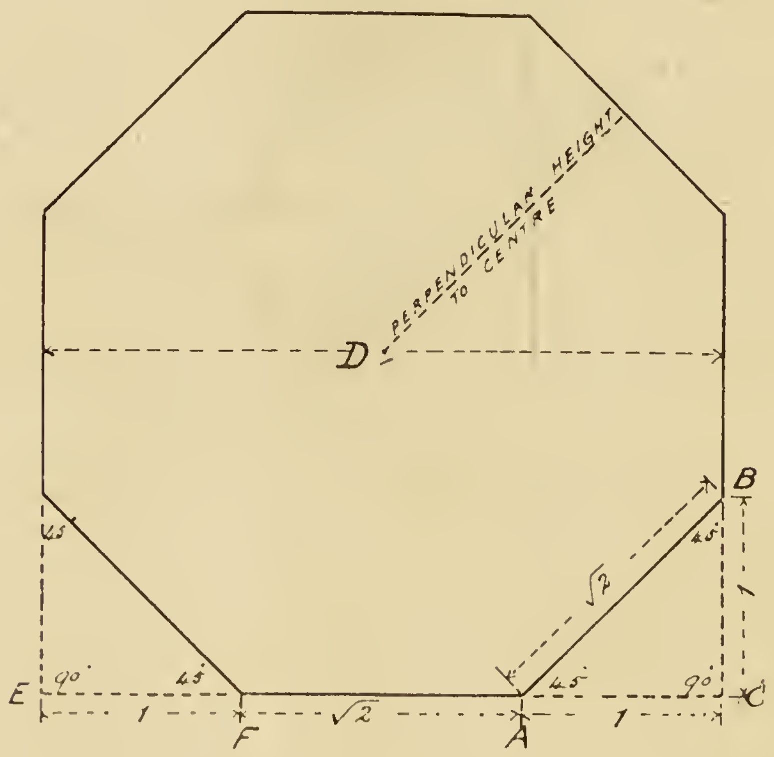


FIG. 35.

Note.—In any right-angled triangle A B C (Fig. 35), the sides A C and C B of which are taken respectively as *unity*, the hypotenuse A B = $\sqrt{1^2 + 1^2} = \sqrt{2} = 1{\cdot}4142136$.

Therefore the straight line E C, which is equal to the diameter of the octagon, consists of the following proportions :—

$$1 + 1{\cdot}4142136 + 1 = 3{\cdot}4142136,$$

of which the length F A (a side of the octagon) equals $\dfrac{1{\cdot}4142136}{3{\cdot}4142136} =$ ·4142136 of the whole E C, *i.e.*, of the diameter D.

To find the area of a circle—

23) *Multiply the square of the diameter by* ·7854.

Or, by another rule—

(24) *Multiply the radius by the circumference, and take half the product.*

To find the radius of a circle, the area being given—

(25) *Divide the area by* 3·1416, *and extract the square root of the quotient.*

To find the area of a circular ring—

(26) *Multiply the sum of the radii by their difference and the product by* 3·1416.

To find the area of a sector of a circle—

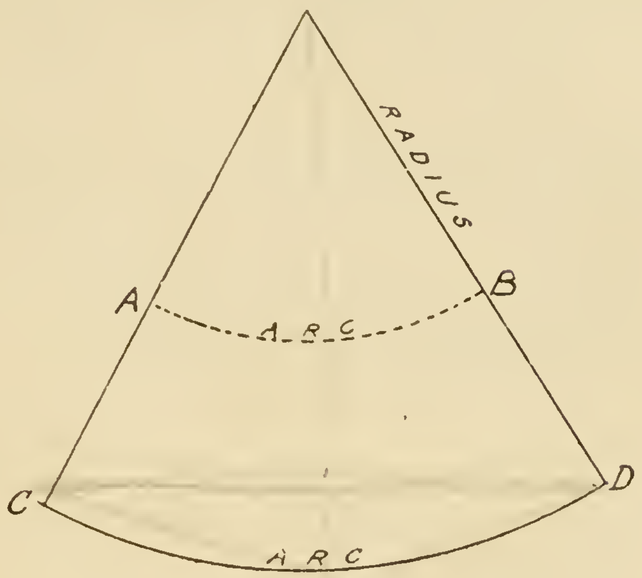


FIG. 36.

(27) *Area of sector* =

$$\frac{\text{Number of degrees in arc} \times \text{area of circle.}}{360}$$

Or, by another rule—

(28) *Area of sector = Length of arc* X $\frac{1}{2}$ *radius.*

To find the area of A B C D in Fig. 36—

(29) *Multiply the sum of the arcs by the difference of the radii and take half the product.*

To find the area of a segment of a circle; the segment being less than a semicircle—

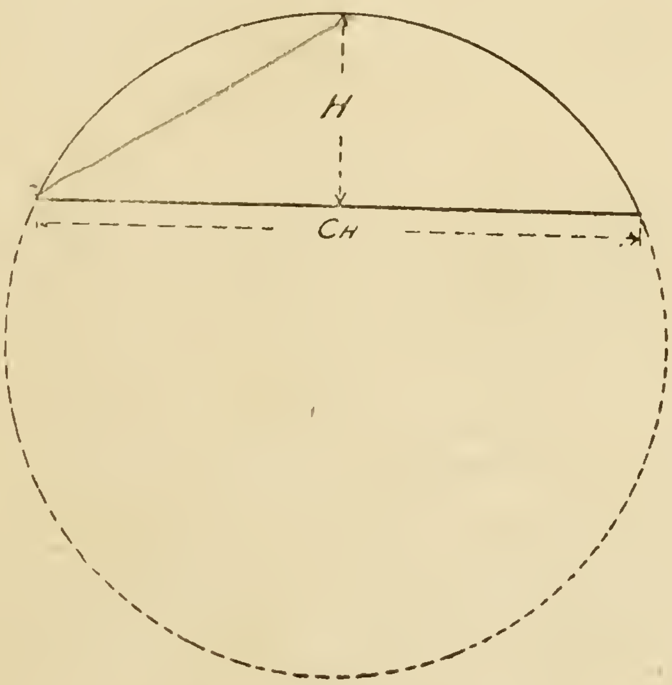


FIG. 37.

(30) *Find the area of the sector which has the same arc, and subtract the area of the triangle formed by the radii and the chord.*

To find area of a segment of a circle—

(31) *To $\frac{2}{3}$ of product of chord and height add the cube of the height divided by twice the chord.*

The rule may be thus expressed—

$$\text{Area} = (\text{Ch} \times \text{H} \times \tfrac{2}{3}) + \frac{\text{H}^3}{2\ \text{Ch}}.$$

Simpson's Rule for calculating areas, or the method of equidistant ordinates. It is required to find the area of the figure A *a* *g* G. Divide the length A G into any *even* number of *equal* parts as A B, B C, C D, &c., and draw the perpendiculars B *b*, C *c*, D *d*, &c. Then to find the *area* of the figure proceed as follows:—

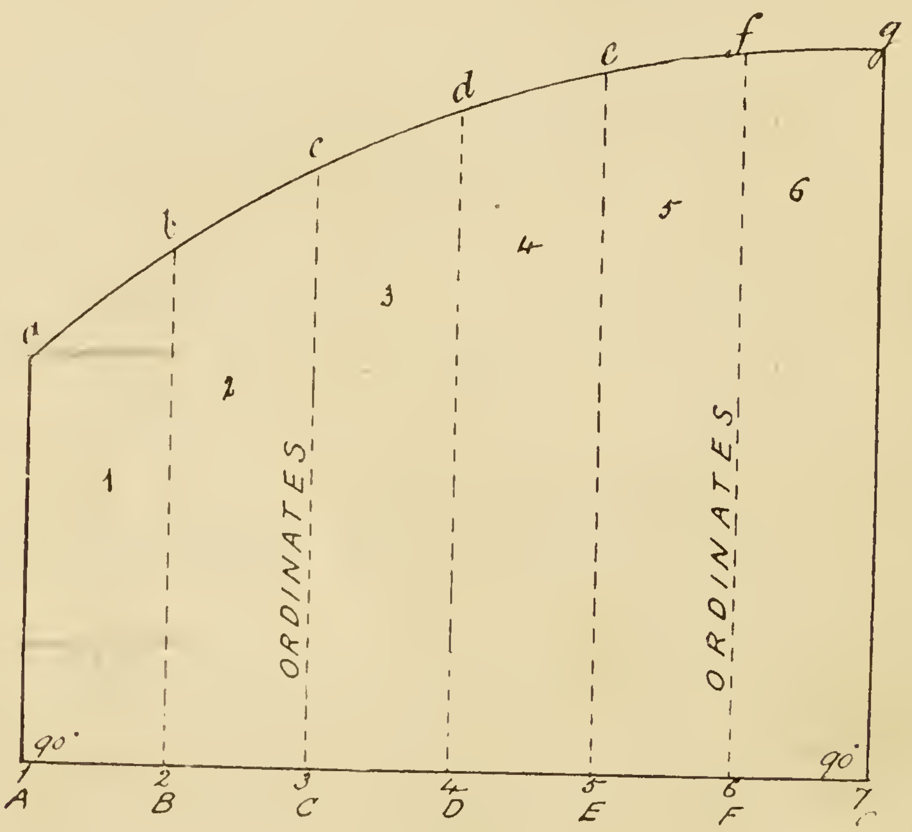


Fig. 38.

(32) "*Add together the first ordinate, the last ordinate, twice the sum of all the other odd ordinates, and four times the sum of all the even ordinates; multiply the result by $\frac{1}{3}$ of the common distance between two adjacent ordinates.*"

The result will not be strictly accurate, but sufficiently so for all ordinary practical purposes; the more *ordinates* used the more accurate will be the result. The rule is used by land surveyors. It is also applicable to areas such as the following Figs 39. and 40.

To find the area of an irregular piece of land, such, for example, as shown in the accompanying sketch (Fig. 41), it is usual in practice to compute it by what is known as *casting*, *i.e.*, by reducing the crooked sides to straight ones, as A B, B C, and C A; the straight line being so drawn that the area of the land *outside* the straight

boundary is equal to the area of that *inside* it; thus leaving the total area practically unaffected, depending, however, of course, upon the accuracy with which the equalising line is drawn.

The *parallel ruler* is also often made use of for reducing crooked boundaries to straight ones; the method is mathematically accurate,

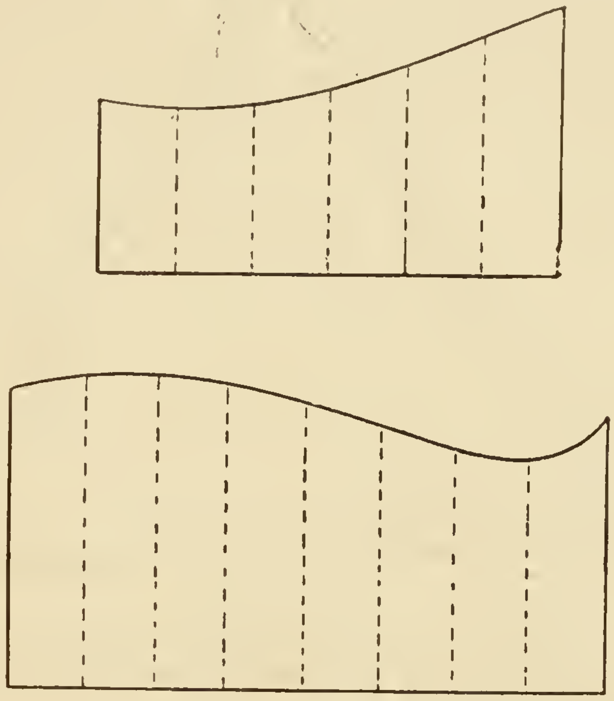


Fig. 39.

and is based upon a familiar proposition of Euclid, viz., that *triangles on the same base, and between the same parallels, are equal* (*vide* Euc. I. 37).

Suppose it is required to reduce the figure A B C D E F (Fig. 42)—which is supposed to be plotted to scale—to a right-angled triangle A B 4 by an equalising line B 4, the method of procedure is as follows[1] :—

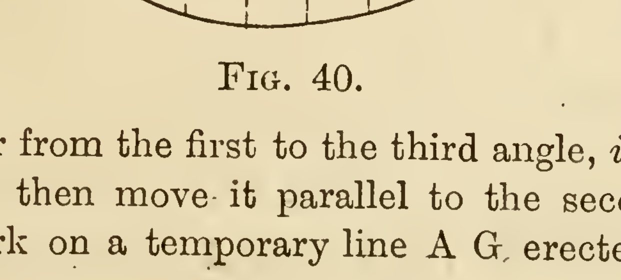


Fig. 40.

(*a*) Lay the ruler from the first to the third angle, *i.e.*, through A E in the figure, and then move it parallel to the second angle at F; make the first mark on a temporary line A G, erected perpendicular to A B.

(*b*) Lay ruler from first mark on the temporary line to the fourth

[1] "Land and Engineering Surveying," by T. Baker, C.E. (Crosby Lockwood and Son.)

D

angle at D, and then move it parallel to the third angle E; now make second mark on the temporary line.

(*c*) Lay the ruler from the last mark on temporary line to the fifth

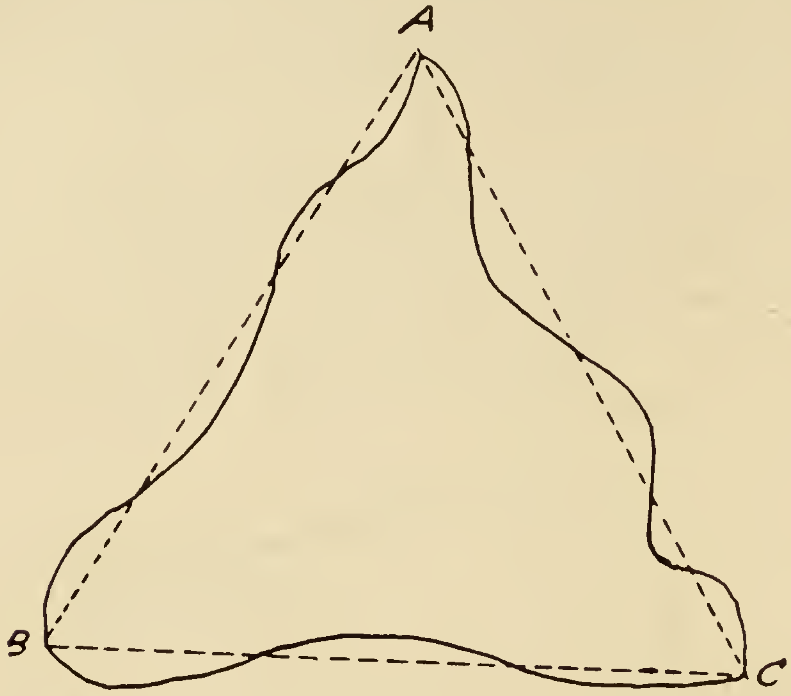


FIG. 41.

angle C; move it parallel to the fourth angle D, and make the third mark on the temporary line.

(*d*) Lay the ruler from the last-named mark on temporary line to

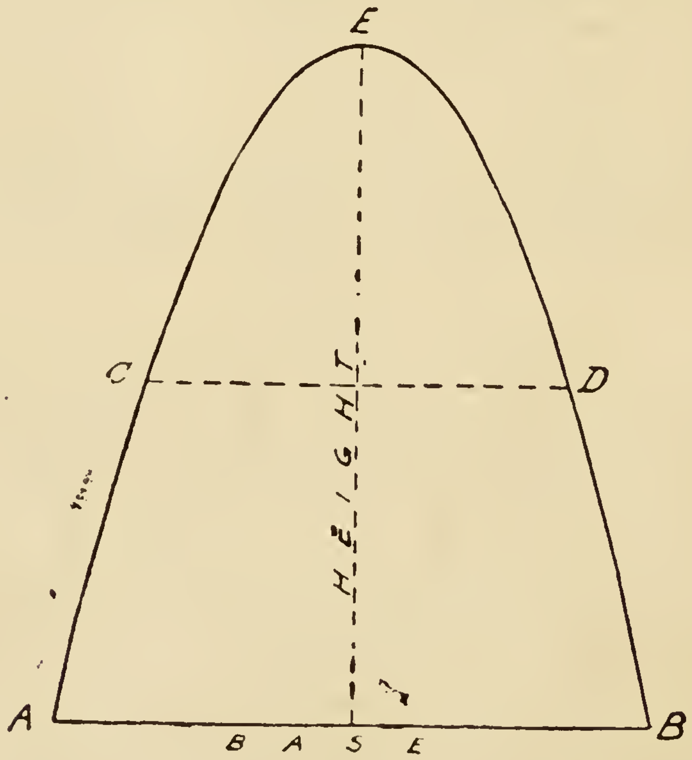


FIG. 43.

the sixth angle B, and move parallel to fifth angle C, making the fourth mark on the temporary line.

Now join B 4, and then the area of the figure A B C D E F = $\frac{1}{2}$ (A B × A 4).

The method is a very useful one, and may be used as a check on a previous calculation; the only source of error likely to arise is that caused by accidental slipping of the ruler or slight inaccuracy of drawing or scaling.

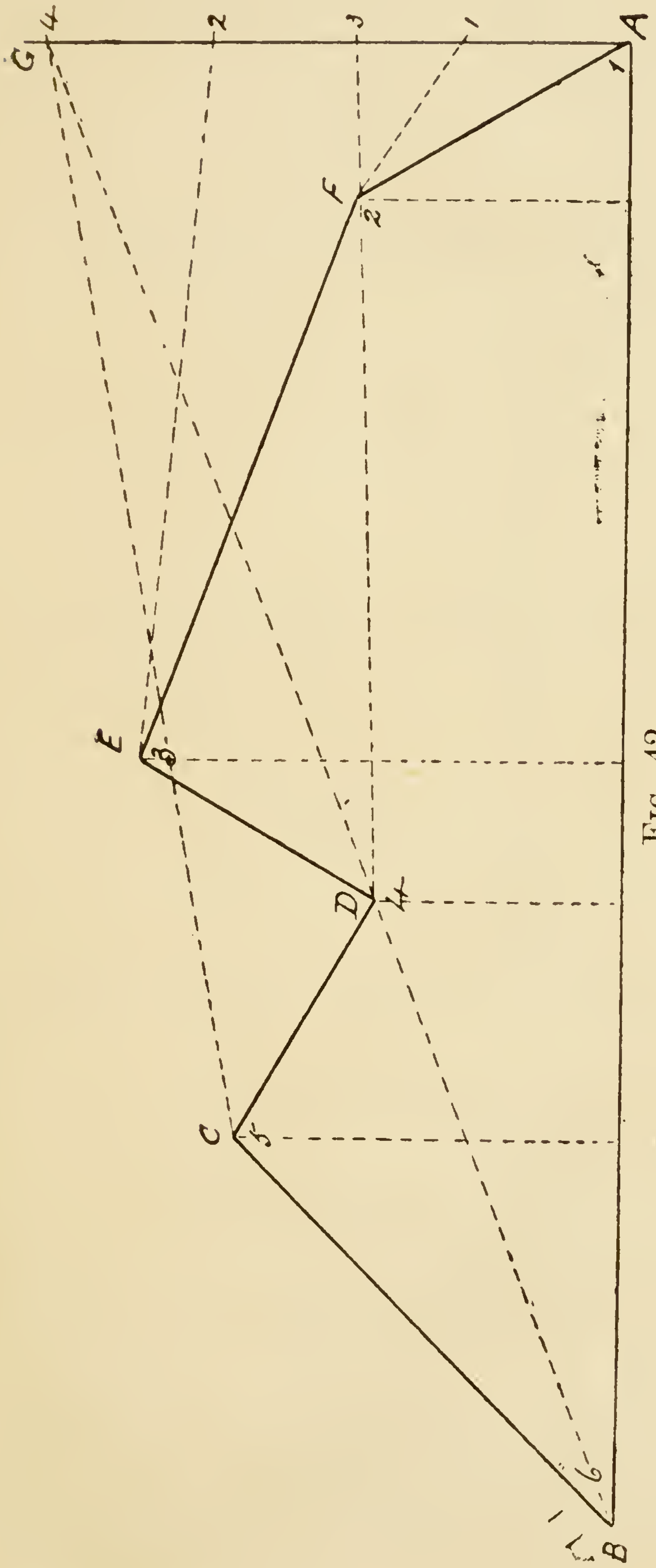


Fig. 42.

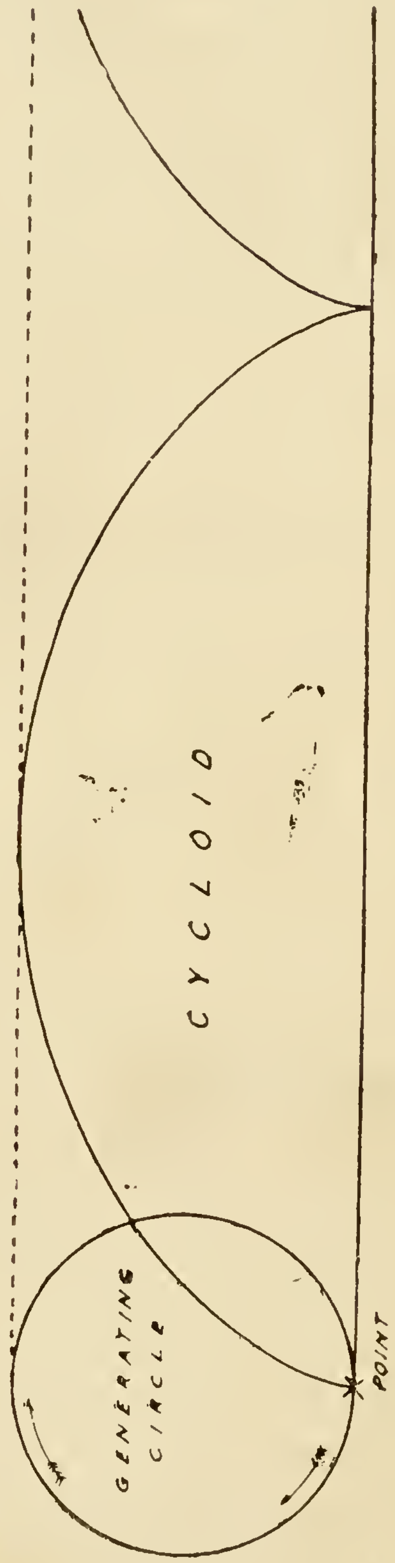


Fig. 45.

To find the area of a parabola (Fig. 43).

(33) *Multiply the base by two-thirds the height.*

The area of the Frustum of a parabola as A B D C (Fig. 43) =

(34) $$\tfrac{2}{3} \text{ height } \frac{\text{base}^3 - \text{top}^3}{\text{base}^2 - \text{top}^2}.$$

The area of an ellipse (Fig. 44)—

(35) = *Transverse axis* × ·7854 *conjugate axis.*

Note:—An ellipse is equal to a circle whose diameter is a mean proportional between the two axes.

The area of a cycloid—

(36) = *Area of generating circle* × 3.

Note:—A *cycloid* is a curve generated by a point in the plane of a circle when the circle is rolled along a straight line, keeping always in the same plane. (See Fig. 45.)

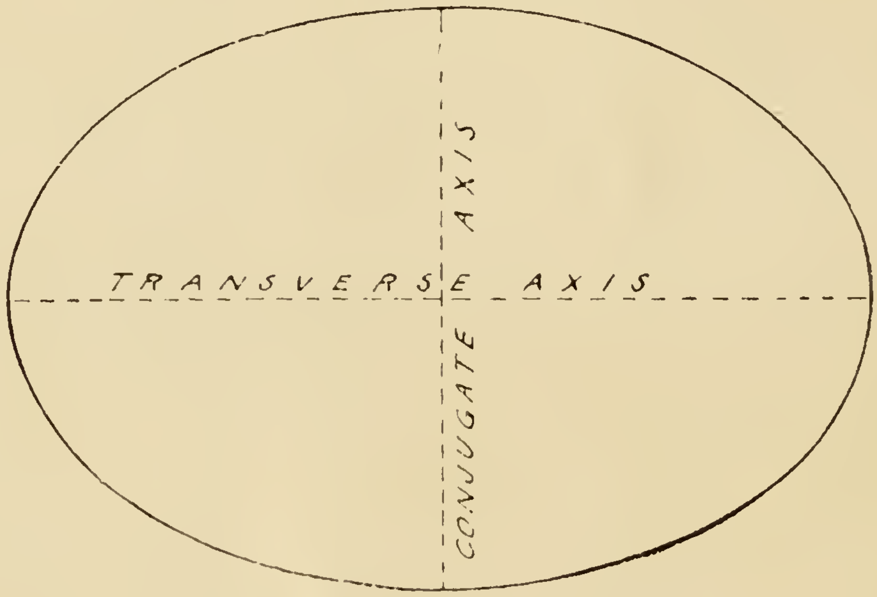


FIG. 44.

The *line* is called the "*directrix*," and the *point* on the circle the "*generatrix*."

ON THE AREAS OF THE SURFACES OF SOLIDS.

The area of the surface of a cylinder—

(37) = *Area of both ends* + (*length* × *circumference*).

The area of the surface of a solid ring—

(38) = *Circumference of a circular section of the ring* × *length of the ring.*

The area of the *curved surface* of a right circular cone—

(39) = *Circumference of the base* × ½ *slant height.*

The area of the curved surface of a frustum of a right circular cone—

(40) = *The sum of the circumferences of the two ends of the frustum* × ½ *slant height of the frustum.*

The area of the surface of a sphere—

(41) $= \textit{Diameter}^2 \times 3\cdot1416$, or

$=$ Circumference $\times$ diameter.

The *surface* of a sphere—

= four times the area of a circle of same diameter as the sphere.

= the area of a circle whose diameter is double that of the sphere.

= the convex surface of the circumscribing cylinder.

To find the area of the curved surface of a zone of a sphere or of a segment of a sphere—

(42) *Multiply the circumference of the sphere by the height of the zone or segment.*

ON THE CALCULATION OF CUBIC CAPACITY OR VOLUME.

The Table of Solid or Cubic Measure may be expressed as follows:—

Inches.		Feet.		Yards.
1,728	=	1		
46,656	=	27	=	1

It will also be convenient to remember that—

1 cubic foot of pure water at 62 deg. Fah. = 1000 ounces = 62·5 lb = ·0278 ton.

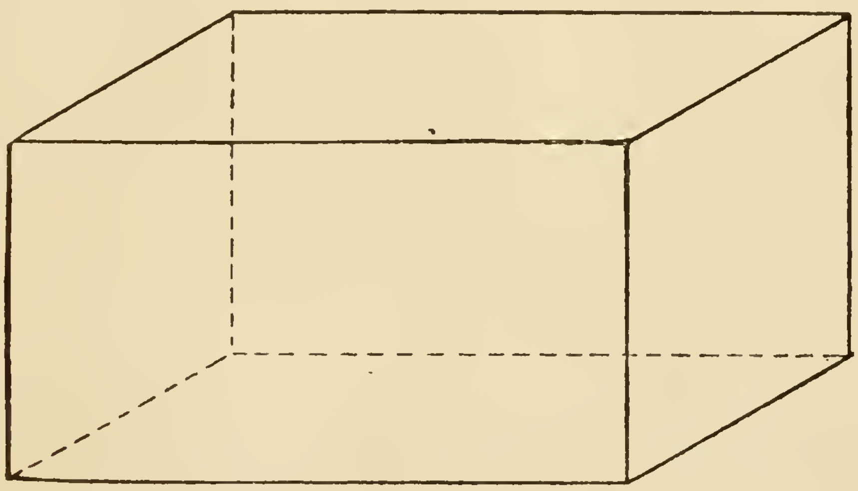


Fig. 46.

1 cubic foot of water = 6·25 gallons.

1 gallon = 10 lbs. = ·16 cubic foot = $277\frac{1}{4}$ cubic inches.

224 gallons = 1 ton = 35·9 cubic feet.

To find the volume of a parallelopiped, a prism, or a cylinder—

(43) *Multiply the area of the base by the perpendicular height, and the product will be the volume.*

Note:—A *parallelopiped* (Fig. 46) is a solid bounded by six parallelograms, of which every opposite two are equal and in parallel planes.

A *prism* (Fig. 47) is a solid whose bases or ends are any similar, equal, and parallel plane figures, and whose sides are parallelograms.

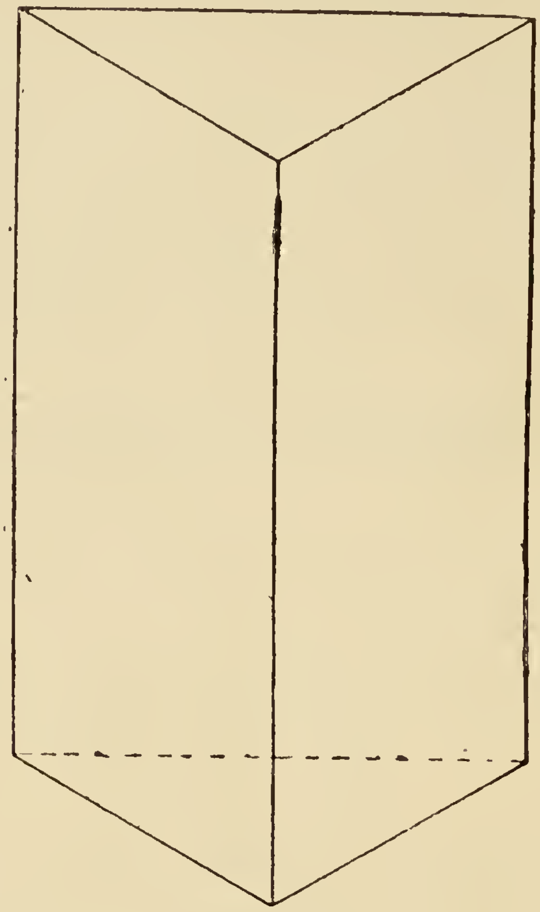


Fig. 47.

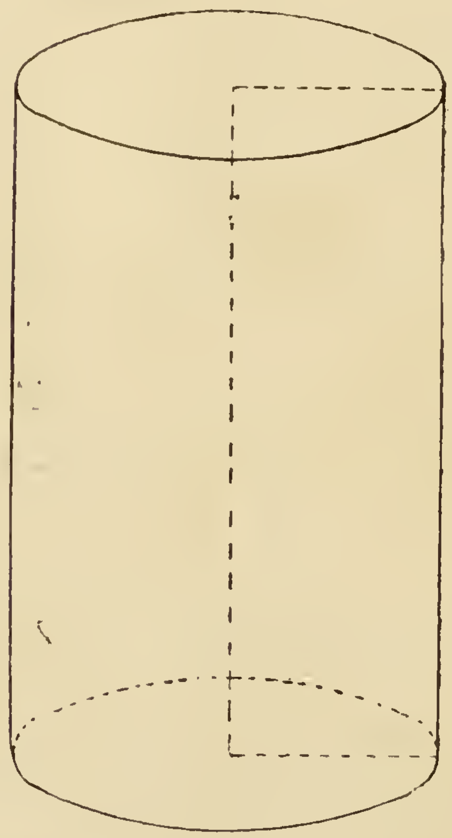


Fig. 48.

A *cylinder* (Fig. 48) is a solid formed by turning a rectangle round one of its sides, which remains fixed.

The above rule holds good, whether the parallelopiped, prism, or cylinder be *right* or *oblique*.

The side of a *cube* equals the cube root of its solidity.

To find the volume of a solid ring—

(44) *Multiply the area of a circular cross section of the ring by the length of the ring.*

To find the volume of a pyramid or a cone—

(45) *Multiply the area of the base by $\frac{1}{3}$ the perpendicular height; the product will be the volume.*

A triangular pyramid is the third part of a prism of the same base and height.

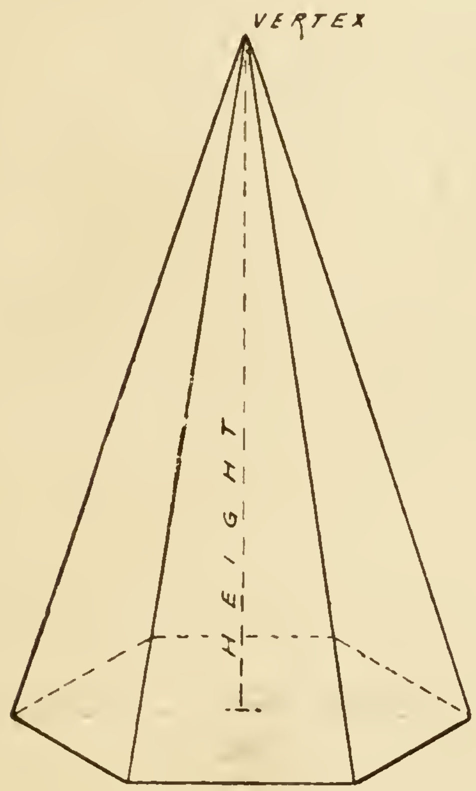


Fig. 49.

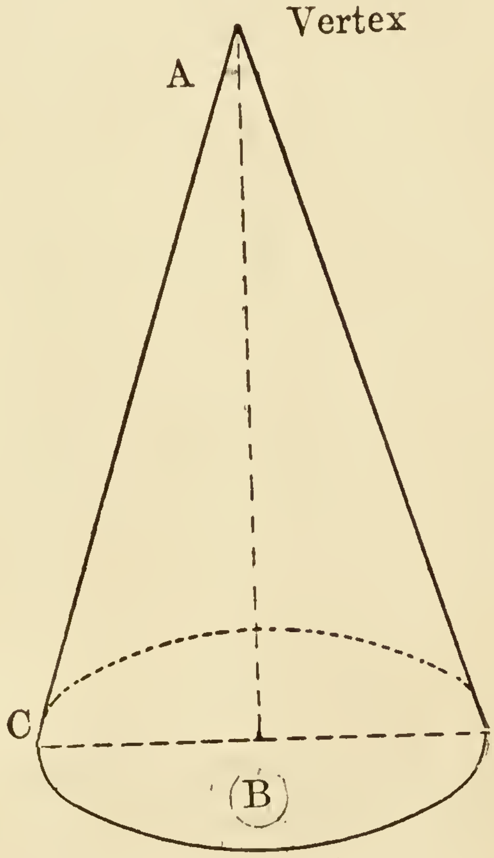


Fig. 50.

The solidity of a cone equals $\frac{1}{3}$ of the solidity of its circumscribing cylinder.

A *pyramid* (Fig. 49) is a solid bounded by three or more triang[les] which meet at a point called the *vertex*, and by one other rectilineal figure called the *base*.

A *cone* (Fig. 50) is a solid produced by turning a right-angled triangle, as A B C, round one of the sides A B which contain the right angle, this side A B remaining fixed. The side A C is called the *slant height*.

To find the volume of a frustum of a pyramid or cone—

(46) *To the sum of the areas of the two ends of the frustum add the square root of their product; multiply this final sum by $\frac{1}{3}$ the perpendicular height; and the product will be the volume.*

This rule may be expressed as follows, where A = area of large end, and a = area of small end.

Volume of frustum $= \frac{1}{3} \text{H} (\text{A} + a + \sqrt{\text{A} \times a})$

Note:—A *frustum* (Figs. 51 and 53) of a solid, such as a pyramid or cone, is that portion of it contained between its *base* and any

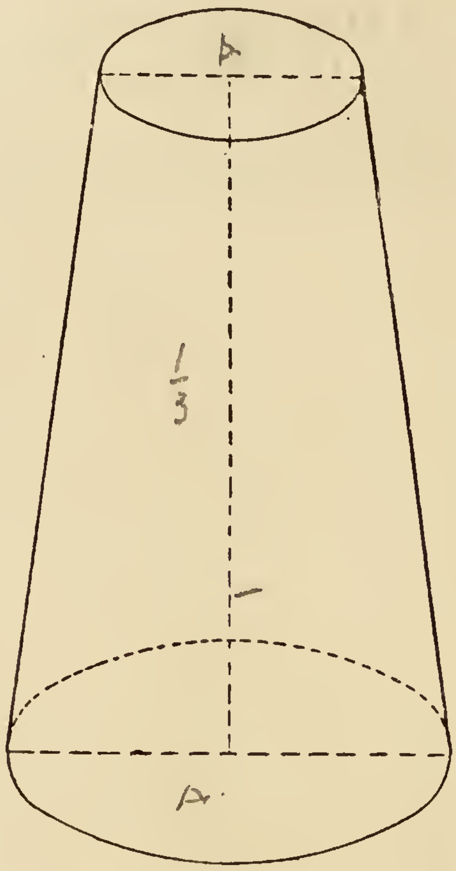


Fig. 51.

plane parallel to the base. The base and the opposite face being known as the *ends* of the frustum.

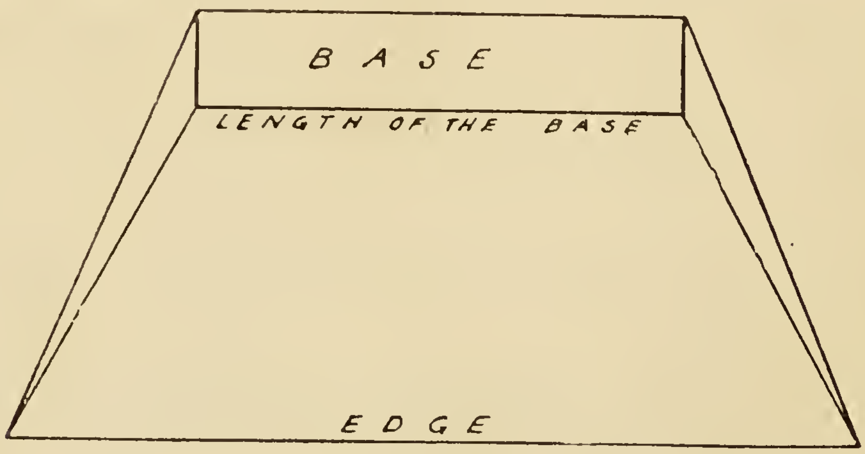


Fig. 52.

To find the volume of a wedge—

(47) *Add the length of the edge to twice the length of the base; multiply the sum by the width of the base and the product by* $\frac{1}{6}$ *of the perpendicular height; the result will be the volume.*

Note :—A *wedge* (Fig. 52) is a solid bounded by five planes; the base is a rectangle, and the two ends triangles, and the other two faces are trapezoids.

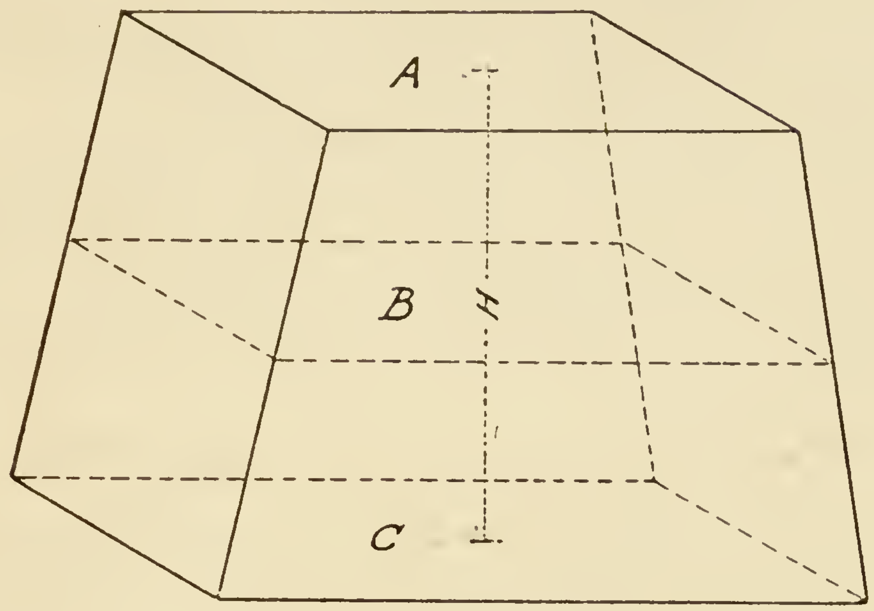


FIG. 53.

To find the volume of a prismoid, or frustum of a wedge—

(48) *To the sum of the areas of the two ends add four times the area of a section parallel to the base and equally distant from both ends; the sum being multiplied by $\frac{1}{6}$ the perpendicular height will give the volume.*

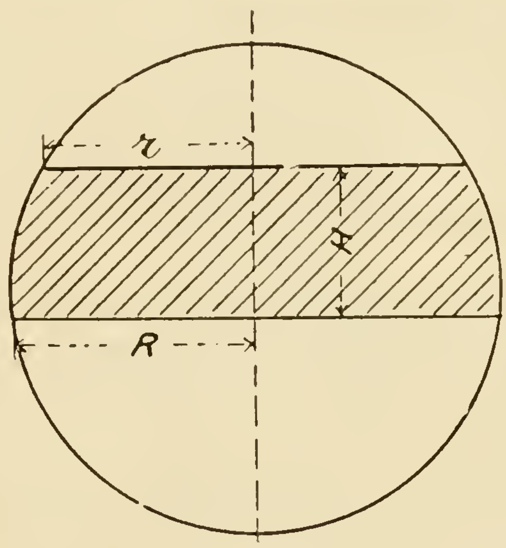


FIG. 54.

Let A, C, and B (Fig. 53) represent the *areas* of the two *ends* and *middle* section respectively, and H = the perpendicular height, then the volume equals—

$$\tfrac{1}{6} H (A + C + 4 B)$$

To find the volume of a sphere—

(49) *Multiply the cube of the diameter by* ·5236, *the product will be the volume.*

Note:—·5236 is one-sixth of 3·1416.

The solidity of a sphere equals two-thirds of the solidity of its circumscribing cylinder.

To find the volume of a spherical shell—

(50) *From the cube of the outer diameter subtract the cube of the inner diameter, and multiply the result by* ·5236.

To find the volume of a zone of a sphere (Fig. 54)—

(51) *To three times the sum of the squares of the radii of the two ends, add the square of the height; multiply the sum by the height, and the product by* ·5236 *; the result will be the volume.*

Let R and r be the radii of the two ends, then the rule may be stated as follows :—

$$\text{Volume} = \left[\left\{ 3\,(R^2 + r^2) + H^2 \right\} H \right] \cdot 5236$$

The volume of a *segment* of a sphere may be found by the same rule, but the radius of one end of a segment is, of course, nothing. The rule, therefore, in this case becomes—

(52) *To three times the square of the radius of the base add the square of the height; multiply the sum by the height, and the product by* ·5236 *; the result will be the volume.*

THE VOLUME OF IRREGULAR SOLIDS.

In practice it is frequently required to be able to ascertain the volume of *irregular solids.* If the solid is of comparatively small size, will sink in water, and will not be injured by it, then it may be immersed in a vessel of a convenient shape for calculation, and the amount of water displaced by the solid will be equal to its own volume.

Should the solid be entirely homogeneous throughout its structure its volume may then be estimated by dividing the weight of a cubic inch of the substance into the weight of the solid, the quotient being the volume in cubic inches.

The weight (in lbs.) of a cubic foot of a given substance = specific gravity of substance × 62·425.[1]

For calculating the volume of large solids such as quantities of earthwork in embankments, cuttings, &c., civil engineers often make use of the following process, based on what is known as Simpson's Rule. (See *ante*, and also "*Calculation of Earthwork.*")

(53) "*Divide the length of the solid into any even number of equal parts; and ascertain the areas of sections of the solid through the points of division perpendicular to the length of the solid. Add together the first area, the last area, twice the sum of all the other odd areas, and four times the sum of all the even areas; multiply*

[1] 62·425 lb. = weight of 1 cubic foot of pure water.

the sum by one-third of the common distance between two adjacent sections."

PRACTICAL EXAMPLES.

A few practical examples embracing the application of some of the foregoing rules will now be worked out in detail. Many of the examples given have been purposely selected from professional examination papers, are of a simple nature, and are chiefly intended to indicate to younger readers the practical application of the subject of mensuration.

(1) Example.—A ladder 40ft. long is placed so as to reach a window 24ft. high on one side of a street, and on turning the ladder over to

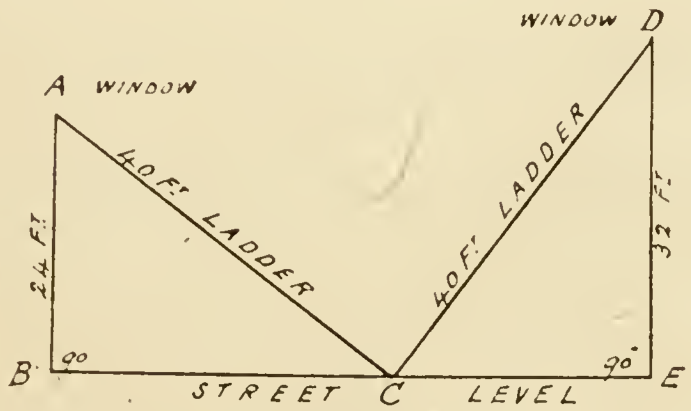


Fig. 55.

the other side of the street it reaches a window 32ft. high ; find the breadth of the street.

It is required to find the two lengths B C and C E.

$$\begin{aligned} BC &= \sqrt{CA^2 - AB^2} \\ &= \sqrt{40^2 - 24^2} \\ &= \sqrt{1600 - 576} \\ &= \sqrt{1024} = 32\text{ft.} \\ CE &= \sqrt{CD^2 - DE^2} \\ &= \sqrt{40^2 - 32^2} \\ &= \sqrt{1600 - 1024} \\ &= \sqrt{576} = 24\text{ft.} \end{aligned}$$

$\therefore$ the breadth of the street is $32 + 24 = 56$ft.

(2) Example.—A footpath goes along two adjacent sides of a rectangle; one side is 196 yards, and the other is 147 yards; find the saving in distance made by proceeding along the diagonal instead of along the two sides.

$$\begin{aligned} AC &= \sqrt{AB^2 + BC^2} \\ &= \sqrt{147^2 + 196^2} \\ &= \sqrt{60025} = 245 \text{ yards.} \end{aligned}$$

The total length round the two sides is 147 + 196 = 343 yards. Therefore the saving in distance is 343 − 245 = 98 yards.

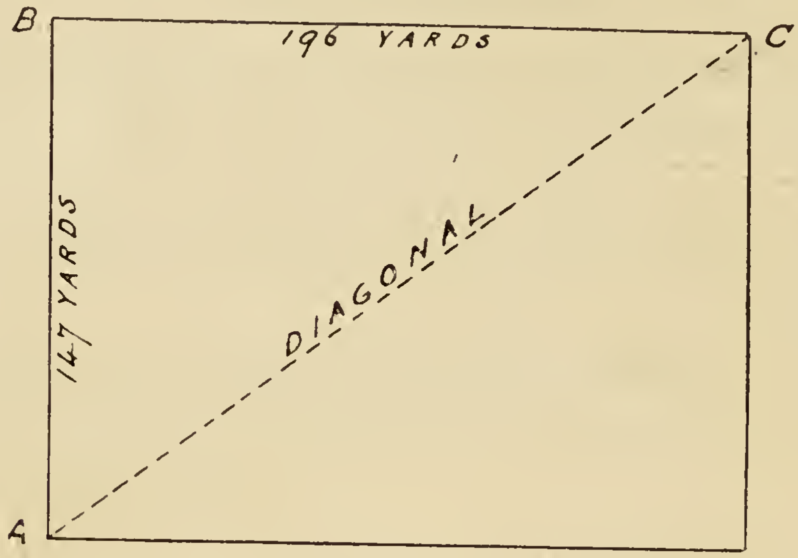


Fig. 56.

(3) Example.—Calculate the cubic capacity of a room 11ft. high, 40ft. by 30ft., and having a semicircular bow 20ft. wide.

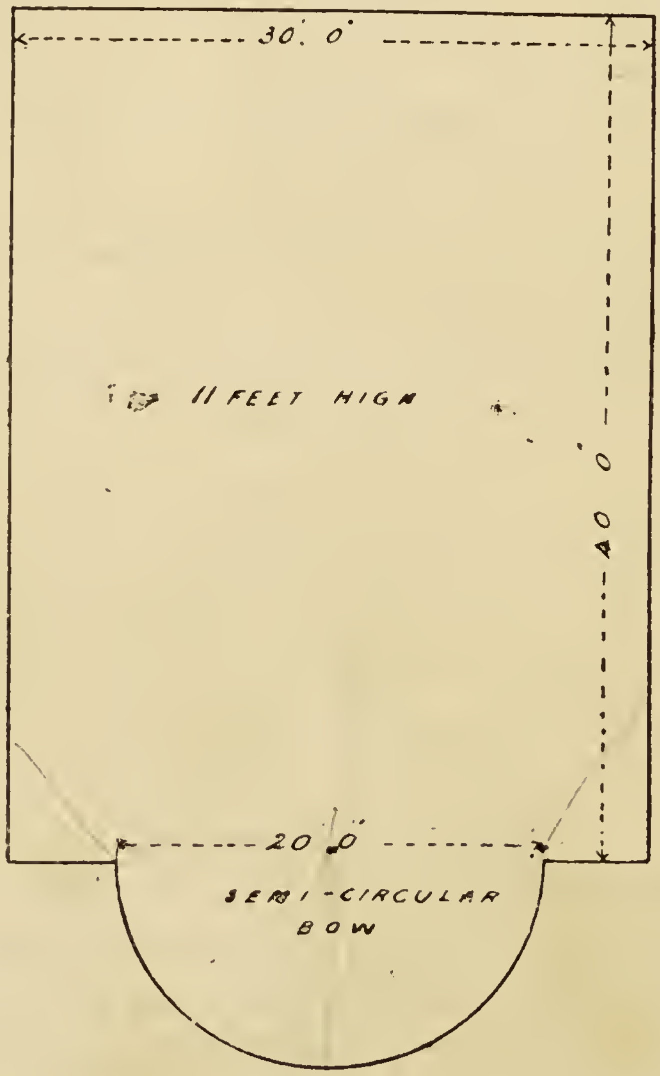


Fig. 57.

Note.—In making any calculation it is always helpful to first make a sketch, figured with the dimensions and particulars given in the problem, as shown in the accompanying diagram.

The floor area of the room = (area of rectangular part + area of bow or semicircular portion) = (40ft. × 30ft.) + $\frac{1}{2}$ (diameter of bow^2 × ·7854) = 1200 + 157·08 = 1357·08 square feet. Cubic capacity = floor area × height = 1357·08 × 11 = 14,927·88 cubic feet.

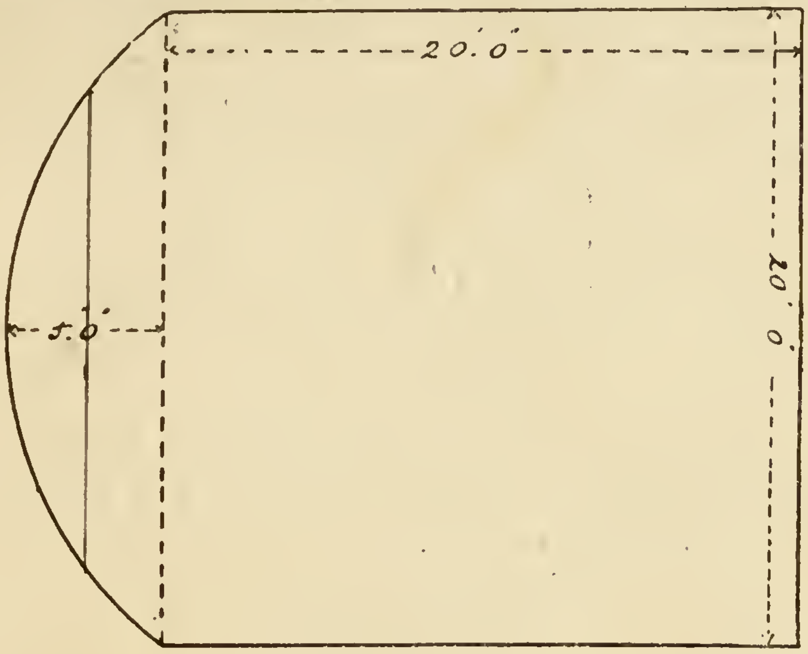


Fig. 58.

(4) Example.—Calculate the cubic contents of a room, as in sketch; one side being the arc of a circle, and the height being 12ft.

The area of segment of a circle may be calculated by the formula—

$$\text{Area} = (\text{Ch} \times \text{H} \times \tfrac{2}{3}) + \frac{\text{H}^3}{2\ \text{Ch}}.$$

$$= (20 \times 5 \times \tfrac{2}{3}) + \frac{125}{40}$$

$$= \frac{200}{3} + \frac{25}{8}$$

$$= \frac{1675}{24} \quad = \quad 69{\cdot}8 \text{ square feet.}$$

Area of rectangular part—

= 20 × 20 = 400·0 square feet.

Total floor area = 469·8

469·8 × 12ft. = 5637·6 cubic feet.

(5) Example.—Calculate the capacity of a cellar 25ft. long by 12ft. wide, having a semicircular roof springing from upright sides 6ft. high.

Capacity of semicircular roof B C D (Fig. 59) = $\frac{1}{2}$ (12^2 × ·7854 × 25) = 1413·72 cubic feet.

Capacity of rectangular portion A B D E = 6 × 12 × 25 = 1800·00 cubic feet.

Total, 3213·72 cubic feet.

(6) Example.—What are the cubic contents of air-space in a room 27ft. 6in. long by 18ft. 9in. broad? The walls are 10ft. high, and the

inside apex of roof 20ft. 4in. high from floor, but the air-space is to be calculated only to a height of 16ft. 9in. from floor.

The first operation in solving this problem will be to find the length A G from the data given.

In the accompanying figure (Fig. 60), representing a cross section of the room in question, H G, B C, and K L are each perpendicular to A D, and the triangles A B C and A H G are therefore "*similar figures*" (see theorem XVII.), so that A G may be found as follows:—

$$\text{B C} : \text{C A} :: \text{H G} : \text{G A}$$

$$\textit{i.e.}\ 10\tfrac{1}{3} : 9\tfrac{3}{8} :: 6\tfrac{3}{4} : \text{G A}$$

$$= \frac{9 \cdot 375 \times 6 \cdot 75}{10 \cdot 33333 +} = \text{G A}$$

$$= 6 \cdot 12\text{ft.} = \text{G A}$$

$$\therefore \text{G L} = 18 \cdot 75\text{ft.} - 2\,(6 \cdot 12\text{ft.}) = 6 \cdot 51\text{ft.}$$

The *total area* of the cross section A H K D F E will therefore be—

Rectangle H L $= 6 \cdot 51 \times 6 \cdot 75 = 43 \cdot 9425$

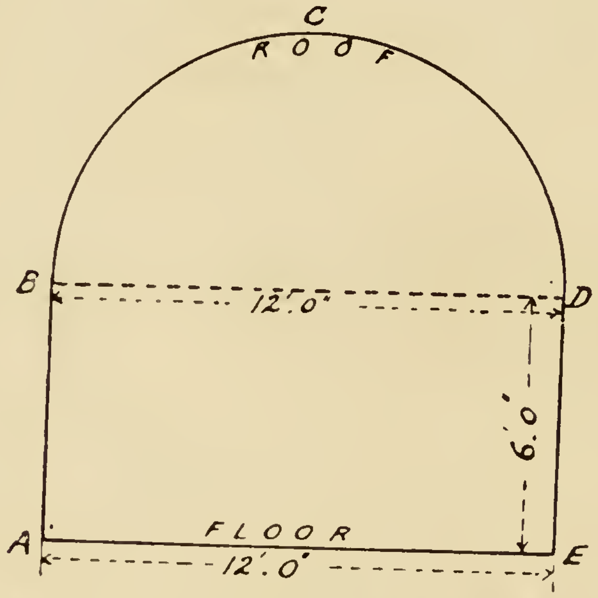


FIG. 59.

Triangles A H G and K L D which are equivalent to rectangle M G	$= 6 \cdot 12 \times 6 \cdot 75$	=	$41 \cdot 3100$
Rectangle A F	$= 18 \cdot 75 \times 10 \cdot 00$	=	$187 \cdot 5000$
Total areas - - - - - - -			$272 \cdot 7525$

Cubic capacity $= 272 \cdot 75 \times 27 \cdot 5$ (length of room)

$= 7500 \cdot 6 +$ cubic feet.

The length G A may also be calculated trigonometrically, as follows:—

First find A B, which equals $\sqrt{\text{A C}^2 + \text{B C}^2}$

$$= \sqrt{9'\,4\tfrac{1}{2}''^{\,2} + 10'\,4''^{\,2}}, \text{ which, reduced to inches}$$

$$= \sqrt{112 \cdot 5^2 + 124^2}$$

$$= 167 \cdot 43\text{in.}$$

$$= 13 \cdot 95 + \text{feet} = \text{A B.}$$

Then,

∵ *The sides of a triangle are proportional to the sines of their opposite angles,*

∴ A B : B C :: sin of angle A C B : sin of angle C A B.

$$= \frac{\text{B C} \times \sin \text{A C B}}{\text{A B}} = \sin \text{C A B}.$$

$$i.e., \text{ sin of angle A} = \frac{\text{B C}}{\text{A B}}.^{1}$$

$$\text{Sin A} = \frac{10'\ 4''}{13 \cdot 95\text{ft.}} = \frac{10 \cdot 3333 +}{13 \cdot 95} = \cdot 7407383.$$

Looking now in a table of sines for ·7407383, we find the nearest to it, without going into *seconds*, corresponds to an angle of 47° 48′[2] =

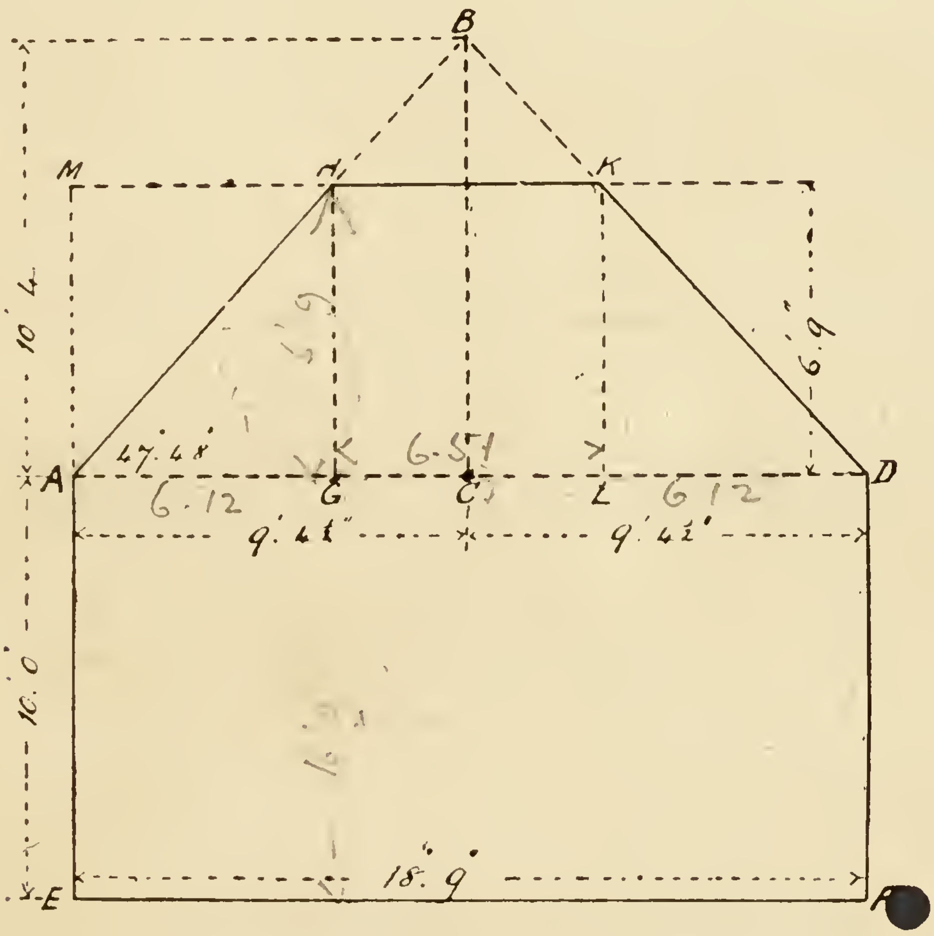


Fig. 60.

angle C A B; and angle A B C or (which is the same thing) A H G = 90° − 47° 48′ = 42° 52′; so that we are now able to state the following proportion—

$$\text{A G} : \text{G H} :: \sin 42^\circ\ 52' : \sin 47^\circ\ 48'$$

$$i.e., \text{ A G} = \frac{\text{G H} \times \sin 42^\circ\ 52'}{\sin 47^\circ\ 48'}$$

[1] In any triangle A B C, the sin of angle A $= \frac{\text{Perpendicular}}{\text{Hypotenuse}}$.

[2] The angle more accurately expressed would be 47° 47′ 39″. 6 +, but the above is sufficient for the present purpose.

$$= \frac{6{\cdot}75 \times {\cdot}6802946^{1}}{{\cdot}7408046}$$

And this, making use of Logarithmic tables,[2] equals—

$= \log.\ 6{\cdot}75 + \log.\ {\cdot}6802946 - \log.\ {\cdot}7408046.$

$= {\cdot}82930 + \bar{1}\,{\cdot}83269 - \bar{1}\,{\cdot}86970.$

$= {\cdot}79229$, the sequence of which is 6·19ft. = G A, which compares sufficiently near with the same length as previously calculated.

Fig. 61.

(7) Example.—Find the cubic capacity of a room 14ft. 9in. in

[1] This is the *natural* sine as distinguished from the *logarithmic* sine, which is the logarithm of the sine of the angle *plus* 10—the *ten* being merely added to eliminate minus quantities from the tables. So that, if a table of log. sines is at hand, the log. sines $\bar{1}$ ·83269 and $\bar{1}$ ·86970 may be found direct upon one reference only to their respective angles in the tables.

[2] C. J. Woodward's "Five-figure Logarithms" have been used above; they give results sufficiently correct for all practical purposes. Chambers' "Mathematical Tables" are an invaluable series, and are universally used.

height as per sketch (Fig. 61). The bow window being 9ft. 3in. high, and on plan a semicircle 8ft. diameter. The bay window 10ft. 9in. high, and on plan a semi-octagon 10ft. diameter.

Cubic Capacity of Bay.—The length of a *side* of the octagon = diameter × ·4142136 as per rule previously given, *i.e.*, 10 × ·4142136 = 4·143 – feet. The area of the semi-octagon may now be found by Rule 21, as follows:—

$\frac{1}{2}$ (4·143 × 5·0 × 8) = 82·86, and the floor area of bay therefore = $\frac{82\cdot86}{2}$ = 41·43 square feet, and the cubic capacity = 41·43 × 10·75 = 445·3 cubic feet.

Cubic Capacity of Bow = $\frac{1}{2}$ (8^2 × ·7854) × 9·25 = 232·5 cubic feet.

Cubic Capacity of Rectangular Room = 22 × 16 × 14·75 = 5192 cubic feet.

Therefore the *total cubic contents* =

	Cubic feet.
Bay	445·3
Bow	232·5
Room	5192·0
	5869·8

(8) Example.—How many gallons of water may be contained in a pipe-drain 6in. in diameter and 30ft. long, and in a vertical soil pipe $3\frac{1}{2}$in. diameter and 20ft. long?

One cubic foot of water = 6·25 gallons.

Therefore the quantity in *pipe-drain* equals,—

$$\cdot5^2 \times \cdot7854 \times 30 \times 6\cdot25 = 36\cdot8 \text{ gallons.}$$

Quantity in *soil pipe*, =

$$\cdot2917^2 \times \cdot7854 \times 20 \times 6\cdot25 = 8\cdot3 \text{ gallons.}$$

Total gallons ... ... 45·1

Questions of this sort may also be very readily worked out by the aid of the following practical rule:—

"The quantity (in gallons) of water contained in a pipe = (diameter in inches)^2 × ·034 × length in feet."[1]

The application of this rule to the present problem gives the following—

$$6^2 \times \cdot034 \times 30 = 36\cdot7 \text{ gallons}$$
$$3\cdot5^2 \times \cdot034 \times 20 = 8\cdot3 \quad ,,$$

Total ... ... 45·0 ,,

[1] See "Drainage Work and Sanitary Fittings" (The St. Bride's Press, Limited, 24, Bride-lane, Fleet-street, E.C.)

(9) Example.—How many inches must the water be lowered in a square tank 12ft. in diameter to fill a 6in. drain 20ft. long ?

The capacity of 6in. drain 20ft. long =

$$\cdot 5^2 \times \cdot 7854 \times 20 = 3\cdot 927 \text{ cubic feet.}$$

The area of the square tank is (12′ × 12′) 144 square feet, and to

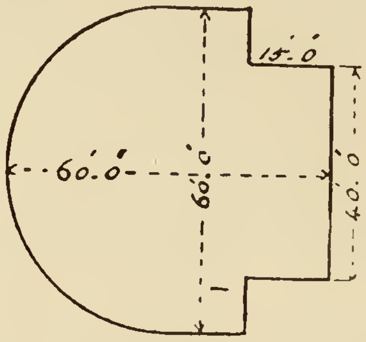


Fig. 62.

obtain in it a capacity equivalent to that of the drain, would require a vertical depth of—

$$\frac{3\cdot 927}{144} = \cdot 0272\text{ft.} = \cdot 3264 \text{ of an inch.}$$

(10) Example.—What is the content in cubic feet of a lecture-room

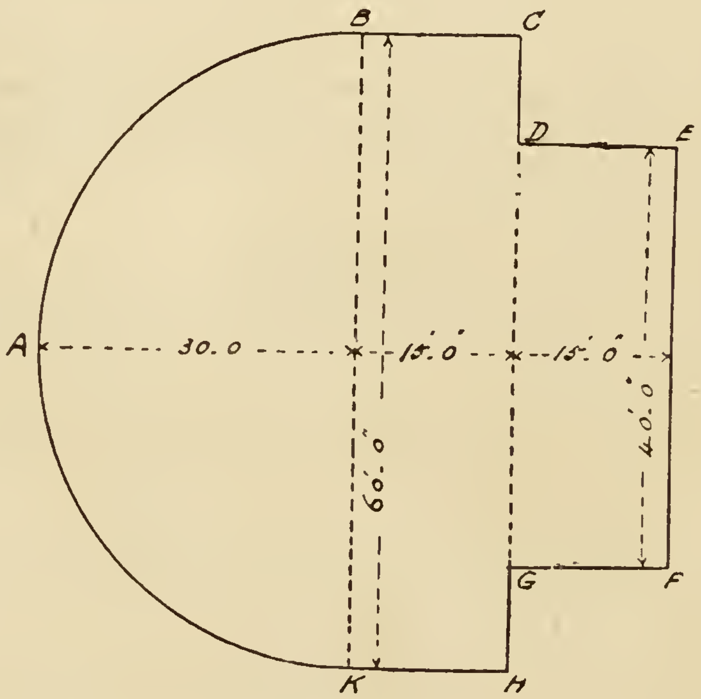


Fig. 63.

of a shape on plan and dimensions as shown in the sketch (Fig. 62), and 35ft. high ?

Floor area =

Area of portion K A B (being a semicircle) = $\frac{1}{2}$ ($60^2 \times \cdot 7854$) = 1413·72 square feet.

Rectangular portion B H = 60 × 15 = 900 square feet.

Rectangular portion D F = 40 × 15 = 600 square feet.

Total floor area = 1413·72 + 900 + 600 = 2913·72 square feet.

Cubic content = 2913·72 × 35 = 101,980 cubic feet.

(11) Example.—How many gallons of water would be contained in a square tank measuring 37ft. along each side of the bottom, the sides sloping outwards at an angle of 45 deg., and the water being 5ft. 6in. deep? Also, what would be the area of the surface of the water?

As the slope of the sides of the tank is 45 deg., the angles A G B and G A B are equal, and the triangle A B G is isosceles, *i.e.*, A B = B G = 5ft. 6in.; and A E therefore equals 5ft. 6in. + 37ft. + 5ft. 6in. = 48ft.

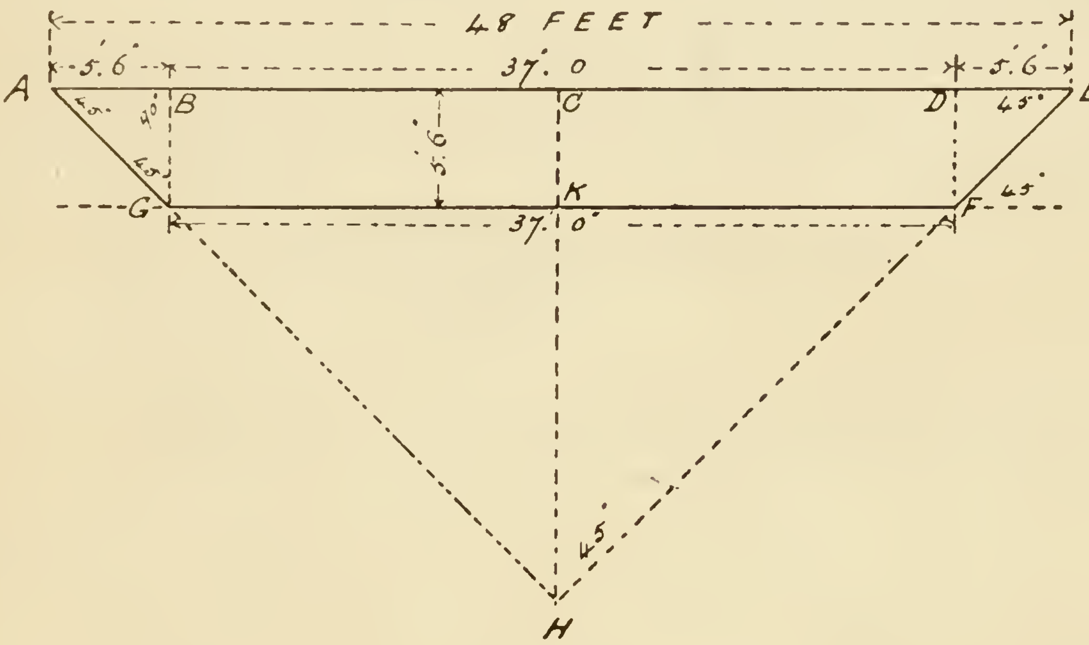


Fig. 64.

In form the tank is a frustum of a pyramid, and its contents may therefore be calculated by Rule 46, which is expressed by the following formula:—

Volume of frustum = $\frac{1}{3}$ H $(A + a + \sqrt{A \times a})$,

where H = height of frustum, and A and a represent the areas of the large and small ends respectively.

Area A = 48ft. × 48ft. = 2304 square feet.

Area a = 37ft. × 37ft. = 1369 square feet.

$$\begin{aligned}
\text{Volume} &= \tfrac{1}{3}\,5\cdot5\,(2304 + 1369 + \sqrt{2304 \times 1369})\\
&= \tfrac{1}{3}\,5\cdot5\,(2304 + 1369 + \tfrac{1}{2}\,(\log 2304 + \log 1369)\\
&= \tfrac{1}{3}\,5\cdot5\,(2304 + 1369 + \tfrac{1}{2}\,(3\cdot36248 + 3\cdot13640)\\
&= \tfrac{1}{3}\,5\cdot5\,(2304 + 1369 + 1776)\\
&= \tfrac{1}{3}\,5\cdot5\,(5449)\\
&= \tfrac{1}{3}\,29{,}969\cdot5\\
&= 9989\cdot8 + \text{cubic feet.}
\end{aligned}$$

The capacity may also be found by calculating the volume of the large pyramid A H E and deducting from it that of the smaller pyramid G H F, thus:—

It will be obvious from an inspection of the figure that C H (the

height of the large pyramid) = C E = 24ft., also that K H (the *height* of the small pyramid) = K F = $18\frac{1}{2}$ft.

The volume of a pyramid = area of base × $\frac{1}{3}$ height.

Therefore, volume of large pyramid = 48 × 48 × $\frac{1}{3}$ (24)
= 18,432 cu. ft.

And volume of small pyramid = 37 × 37 × $\frac{1}{3}$ ($18\frac{1}{2}$) = 8,442 cu. ft.

Total volume = 9,990 cu. ft.

(12) Example.—Find the wetted perimeter and sectional area of sewage flow in a circular sewer, 6ft. in diameter, and running $\frac{1}{3}$ full.

Find length B C, which

$$= \sqrt{\mathrm{B\,A}^2 - \mathrm{A\,C}^2}$$
$$= \sqrt{3^2 - 1^2} = \sqrt{8} = 2{\cdot}8284271\text{ft.}$$

Angle B A C—

$$\sin \angle \mathrm{B\,A\,C} = \frac{\text{Perpendicular B C}}{\text{Hypotenuse A B}}$$
$$= \frac{2{\cdot}8284271}{3} = {\cdot}9428090,$$

which is the sine of an angle of 70° 31′ 44″ = 70·5288 deg.

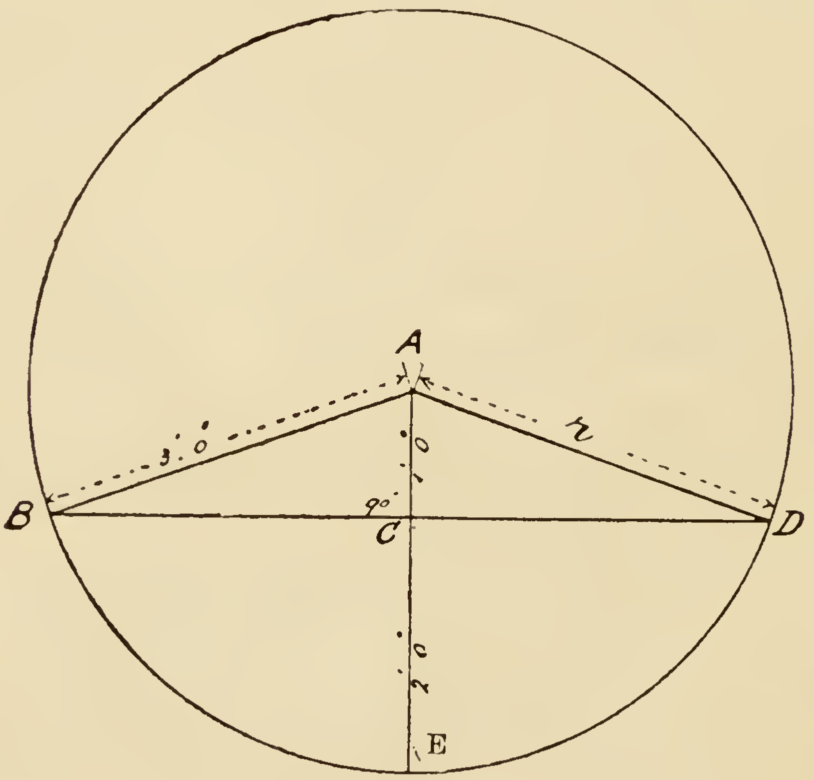


FIG. 65.

Length of Arc B E (by rule previously given).
= 70·5288 × ·017453 × 3 = 3·698ft.

∴ the arc of the segment B E D = 2 (3·698) = 7·396ft., which is the wetted perimeter.

Area of Sector A B E D—
= length of arc × $\frac{1}{2}$ radius
= 7·396 × 1·5 = 11·094 square feet.

Area of Triangle A B D—

$= BC \times AC = 2{\cdot}8284 \times 1 = 2{\cdot}8284$ square feet.

$\therefore$ *Area of Segment* B E D $= 11{\cdot}094 - 2{\cdot}8284 = 8{\cdot}2656$ square feet, which is the sectional area of sewage flow.

A useful practical rule for readily calculating the sectional area of flow and wetted perimeter or segment and arc in the above case, where the flow is one-third full, is as follows*:—

Segment Area $= {\cdot}229\ D^2$

$= {\cdot}229 \times 6^2 = 8{\cdot}244$ square feet.

Length of Arc $= 1{\cdot}23\ D$

$= 1{\cdot}23 \times 6 = 7{\cdot}38$ft.

In connection with the last problem and figure, the following rules may also be of service in calculating the length of the Chord, Arc, and the Area of the Segment:—

The *Chord of a Circle* subtending angle A°.

$= 2\,r\,.\,\sin \frac{A}{2}$

$= 2\,(3)\,.\,\sin < BAC$

$= 6 \times {\cdot}9428090$

$= 5{\cdot}656854 = BD$, *i.e.* $BC = 2{\cdot}828427$ft. as before calculated.

The Arc of a Circle subtending angle A°.

$= \frac{A}{180} \times \pi\, r$

$= \frac{141{\cdot}0576}{180} \times 3{\cdot}1416 \times 3$

$= \log.\ 141{\cdot}0576 - \log.\ 180 + \log.\ 3{\cdot}1416 + \log.\ 3$

$= 2{\cdot}1493965 - 2{\cdot}2552725 + {\cdot}4971509 + {\cdot}4771213$

$= 3{\cdot}1236687 - 2{\cdot}2552725$

$= {\cdot}8683962$

$=$ log. of $7{\cdot}385 +$ feet.

The Area of a Circular Segment, B E D,—

$= \pi\, r^2 \frac{A}{360} - \frac{r^2 \sin A}{2}$

$= \left(3{\cdot}1416 \times 3^2 \times \frac{141{\cdot}0576}{360}\right) - \left(\frac{3^2 \times {\cdot}6286420\dagger}{2}\right)$

$= 11{\cdot}078 - 2{\cdot}828$

$= 8{\cdot}25$ square feet.

(13) Example.—Calculate the capacity in gallons of a rectangular tank 215ft. × 215ft., and 12ft. deep. Also, the pressure in pounds per

---

* For other similar rules applying to *egg-shaped* and *ovoid* sections running $\frac{1}{3}$, $\frac{1}{2}$, $\frac{2}{3}$, and full, see "Drainage Work and Sanitary Fittings," by William H. Maxwell (The St. Bride s Press).

† Sin A = Sin (180 − A), *i.e.* Sin 141° 3′ = Sin (180° − 141° 3′) = Sin 38° 57′ = ·6286420.

square foot on the bottom, and the total internal pressure upon one side of the tank.

The cubic capacity of tank = 215 × 215 × 12 = 554,700 cubic feet.

And one cubic foot = 6·25 gallons.

∴ 554,700 cubic foot × 6·25 = 3,466,875 gallons total capacity.

The hydrostatic pressure upon the bottom of the tank will be directly proportional to the *head* of water, which in this case is 12ft.,

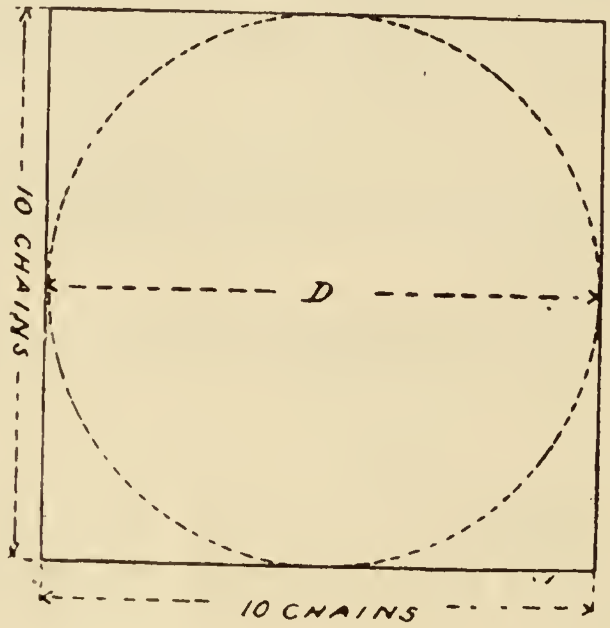


FIG. 66.

and the pressure per square foot will therefore equal the weight of a column of water whose base is 1 square foot, and whose height is 12ft., *i.e.*—

1 square foot × 12ft. (head) × 62·5 lbs. (the weight of 1 cubic foot of water) = 750 lbs.

The internal pressure on one side will be equal to the weight of a column of the liquid, whose base equals the area pressed, and whose height equals the depth of the centre of gravity of the area pressed below the surface of the liquid.

The area of one side = 215 × 12 = 2580 square feet.

The side of the tank is a rectangle, and its centre of gravity will therefore be at its geometrical centre, *i.e.*, 6ft. below the surface of the water.

And, ∵ one cubic foot of water = 62·5 lbs.

∴ Total internal pressure on one side of the tank equals,

$$2580 \times 6 \times 62{\cdot}5 = 967{,}500 \text{ lbs.}$$

(14) Example.—Calculate the area (in *acres*, *roods*, and *perches*) and the diameter (in chains) of the largest circle that can be laid out in a square plot of land 10 acres in extent.

Area of square plot of land (Fig. 66) = 10 acres = 100 square chains.

The side of the square $= \sqrt{100} = 10$ chains, which will also be the *diameter* of the circle.

The *area of the circle* = diameter^2 × ·7854 =

$$10^2 \times \cdot 7854 = 78\cdot 5400 \text{ square chains}$$
$$= 7\cdot 854 \text{ acres}$$
$$4$$
$$\overline{3\cdot 416} \text{ roods}$$
$$40$$
$$\overline{16\cdot 640} \text{ perches}$$

The area of the circle therefore is 7 acres 3 roods 16·6 perches.

(15) Example.—A rectangular piece of land, 3 acres in area, the sides of which are as 1 to 5, is to be divided into 40 equal building

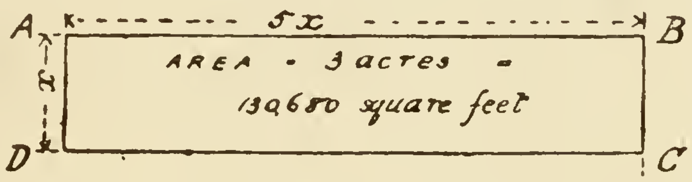


FIG. 67.

plots, having frontages to the long side—calculate the frontage, in feet, of each.

Let x = length of short side A D, then $5x$ = length of long side A B, and, the *area* of the land =

$$5x \times x = 130{,}680 \text{ square feet (\textit{i.e.}, 3 acres).}$$
$$= 5x^2 = 130{,}680$$
$$= x^2 = \frac{130{,}680}{5} = 26{,}136$$
$$= x = \sqrt{26{,}136} = 161\cdot 6 + \text{feet} = \text{the length of A D.}$$

And the length A B therefore

$$= \frac{\text{Area}}{\text{A D}} = \frac{130{,}680}{161\cdot 6} = 808\cdot 6 + \text{feet.}$$

∴ the frontage of each plot

$$= \frac{808\cdot 6}{40} = 20\cdot 2 + \text{feet.}$$

(16) Example.—A 3-chain scale was used for calculating the area of a piece of land on a plan drawn to a scale of 4 chains to 1in.; the result was 8 acres 3 roods. What was the correct area?

The computed *area* will be to the true *area* as the *square* of three is to the *square* of four, *i.e.*:—

$$3^2 : 4^2 :: 8 \text{ ac. } 3 \text{ rd.} : \text{true area.}$$
$$9 : 16 :: 35 \text{ rd.} : \text{true area.}$$
$$= \frac{16 \times 35}{9} = 62\cdot 2 \text{ rds.}$$
$$= 15 \text{ acres } 2\cdot 2 \text{ roods.}$$

ON THE CALCULATION OF QUANTITIES OF EARTHWORK.

Another example of the practical utility of the science of "*mensuration*" is found in its application to the calculation of quantities of

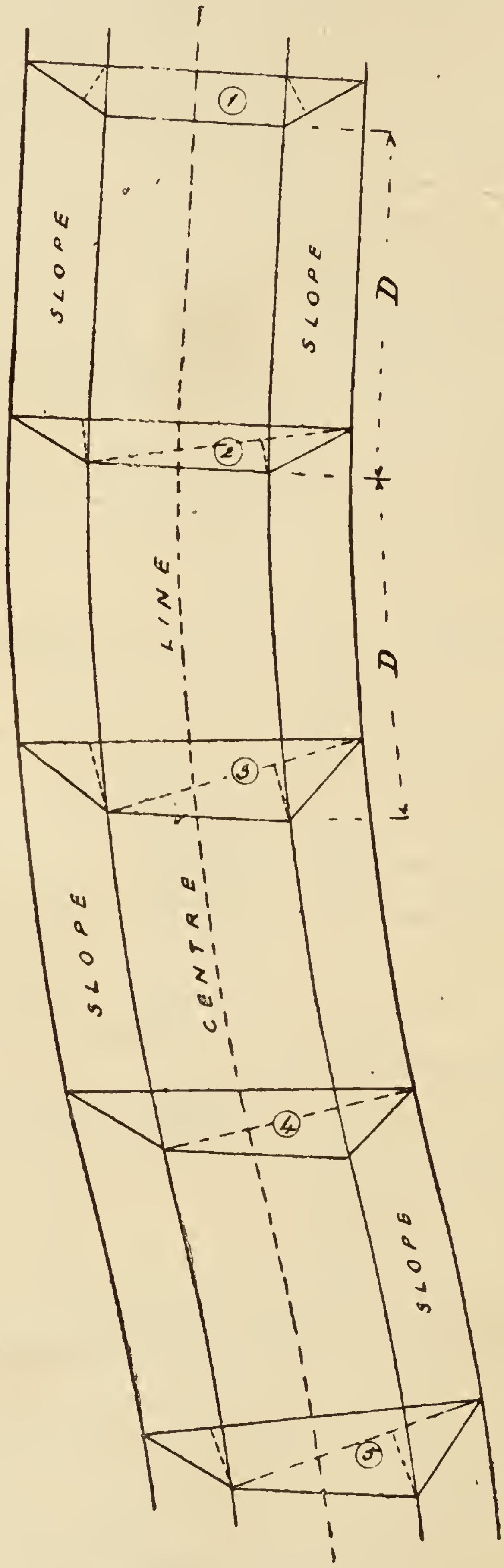


FIG. 68.

earthwork, such as for sewers, drains, canals, railways, embankments, cuttings, &c.

These quantities may be accurately calculated by dividing the various solid masses into definite geometrical forms, such as pyramids, prismoids, wedges, &c., the cubic capacities of which are obtained by the foregoing rules of mensuration.

A less accurate, but more expeditious method, is to divide the given length of cutting or embankment into a number of equi-distant perpendicular transverse sections, and then calculate the cubic capacity between each pair by multiplying the half sum of the pair of sectional areas by the distance between them; or, the area of a single middle section may be taken (assuming it to be a fair average section of the length to be calculated), and multiplying by the distance as above.

The method of taking the average of the top and bottom *breadths*, and average *depths* at various points, and multiplying the product of these by the *length* of the work, is also sometimes adopted, but is likely to lead to erroneous results.

A more accurate method is to find the areas of a number of equi-distant perpendicular transverse sections,[1] and then proceed with them as if they were so many equi-distant ordinates; the result being the cubic capacity between the first and last cross sections.

The rule for calculating irregular figures by means of equi-distant ordinates is known as "*Simpson's Rule*," and is as follows:—

Divide the given length into any *even* number of *equal* parts; then, "*add together the first ordinate, the last ordinate, twice the sum of all the other odd ordinates, and four times the sum of all the even ordinates; multiply the result by one-third of the common distance between two adjacent ordinates.*"

To put this in another form—

Let A = sum of areas of the first and last ordinates (*i.e.*, of cross-sections Nos. 1 and 5, Fig. 68).

Let B = sum of all other *odd* ordinates (cross-section No. 3).

Let C = sum of all the *even* ordinates (cross-sections Nos. 2 and 4).

Let D = common distance between two adjacent ordinates.

Then $(A + 2B + 4C) \times \frac{1}{3} D$ = cubic contents.

The above method, though very fairly correct if the cross-sections are taken at short intervals, is a somewhat tedious one, and, in *practice*, would involve too much time, as all the sections require to be plotted to calculate their respective areas.

The *Prismoidal Formula* is frequently used, and is as follows:— "*The area of each end added to four times the middle area, and the sum multiplied by the length divided by six, will give the solid content.*"

[1] The result will be more accurate the greater the number of sections taken; and the sections should be nearer together on uneven ground than on level.

That is :—

$$\frac{[\text{Sum of areas of both ends} + (\text{middle area} \times 4)] \times \text{length}}{6}$$

= solid content.

"To measure the solidity over large areas of irregular depth, divide the surface into triangles, and multiply the area of each by $\frac{1}{3}$ of the sum of the depths taken at the angles, and the result will equal the solidity.

"The surfaces of the triangles must be true planes, or they must be taken so small as to approximate to true planes. J. W. Smith suggests the division of the area into parallelograms; then the cubic contents = $\frac{1}{4}$ (sum of depths at the angles) × (Area of Parallelogram)." [1]

[1] Molesworth's "Engineer's Pocket-book." Hurst.